Movement on Board for Mathematics Teaching

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Abstract:

This curriculum unit uses animation and transition techniques learned in the TIP seminar to enhance PowerPoint presentations presented for Calculus and Algebra 2 lessons. The animations are engaging for students and the lesson format supportive of visual learners "whole picture" approach to learning. The student work includes transformation flipbook and two introductory integration lessons using the principles of Riemann sums.

Keywords: Animation, PowerPoint, Riemann sums, Calculus, Algebra 2,visual learning style

Content Objectives

Introduction

In my seminar, "A Visual Approach to Learning Math", we explored techniques to create animations in all levels of mathematics which "develop fun and engaging approaches to our classroom math lessons". Specifically, we discussed how to animate mathematics while being mindful of aesthetics – making certain the graphics are pleasing to the eye, exciting to watch and appropriate for the medium of choice. We learned how to draw and animate mathematics using PowerPoint morph transitions and animation at a variety of levels joining art, numbers, and math. The seminar showed us how to break complex problems into simpler ones and identify questions students might ask and then use that knowledge to get to the core of the idea using animation. I will utilize seminar material about animation to create high school math lessons emanating from our discussions which bring mathematical concepts to life using movement. This curriculum unit focuses on math content and by using the techniques learned in the seminar as a template, transforming lesson delivery to include animation and movement of screen elements. These lessons are designed so that students in Algebra 2 and Calculus classes can have a visual experience with mathematical content rather than only experiencing it in a static 2-dimensional space. The objective is to create visually appealing lessons which better engage students, especially visual-spatial learners.

Problem Statement

I teach high school math at The Philadelphia High School for Girls, a school with a rich history of academic excellence. Founded in 1848 to "prepare teachers for the common schools of Philadelphia," Girls' High, as it is affectionately known, was the first municipally supported secondary school for girls in the United States and was called the Girls' Normal School. In 1893, the Philadelphia High School for Girls separated from the Girls' Normal School, and the foundation for today's college preparatory curriculum was laid. The school continues its legacy as a school for academically talented girls, providing young women with outstanding opportunities for scholarship, leadership, and service. Its motto, "Vincit qui se vincit" (He conquers who conquers himself), is a key centering point for our students maturing into young woman of purpose and honor (Cutler, 2012).

Research is clear that there is no one clear learning style for BIPOC student or any other group of leaners. There are studies which guide our teaching and inform educators on key elements which are helpful for reaching more students.

By the time my students enroll in high school they have taken high school preparatory courses and have experienced a modicum of success. They are a group of highly able learners who bring energy and a thirst for learning to education. When they interact with the academic content and each other they question and engage is high level conversations for both the content and the work they are given. Even with their strong educational background, my students struggle to apply and extend the mathematical concepts to real life word problems.

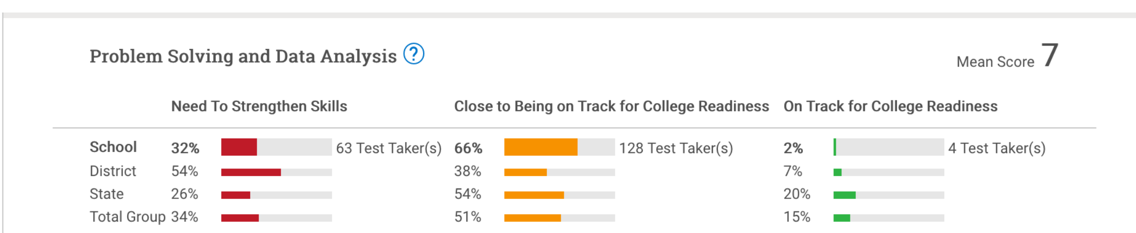
While strong math students in previous settings, across all grades, my students' math skills are often rudimentary and reflect rote learning. Their math knowledge consists of algorithms and formulas of which they have little practical knowledge, and they do not see how those skills apply to real world applications. My students read the problem and focus narrowly on the question. They focus on remembering "the right way," the "formula" or "algorithm," or "the prior lesson" required to solve it. While my students have solid math skills and a strong grasp of math fundamentals, they lack the ability to apply these skills to new math content and to extend that knowledge to think critically about the math. My students fail to learn deeply and lack the ability to apply the recent skill later when it is applied to a more complex problem. As a teacher I experience these student gaps despite published stories trumpeting significant progress in student achievement by students in Philadelphia School District and the United States.

In February 2020, the School District of Philadelphia announced the fourth consecutive year of improvement citywide on the School Progress Report (SPR). The School District analysis said, "After years of investments, we are seeing increases, particularly in climate and (academic) progress". "The SPR is the District's primary tool to measure progress towards the Action Plan 3.0 anchor goals on grade-level literacy and college and career readiness. The SPR evaluates schools in multiple areas, including student achievement, student progress, school climate, and for high schools, college, and career readiness. It is a transparent, actionable way to gauge the progress and achievement of a school. Schools across the city also made improvements in the individual domains used to calculate SPR scores, which also matter greatly to student success. Since 2014-2015, the percentage of schools have increased in major domains: 58 percent, in Achievement; 67 percent, in Progress; 82 percent, in Climate; and 60 percent of schools serving grades nine through 12, in College and Career Readiness." Girls' High's SPR also improved though not as dramatically as those experienced citywide. For the progress category which measures growth on standardized assessments and progress toward graduation, Girls' High scored 84% which corresponds to the "model" category, the highest of the three overall rating categories. Of the fourth categories measured this was the only category to earn the "model" rank.

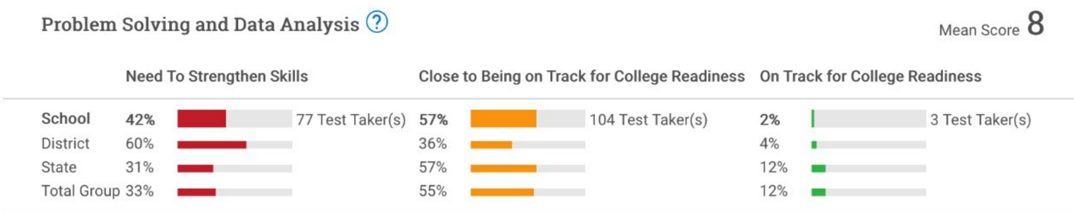
This is great news and would suggest that students entering high school are better prepared and ready to show significant gains in all subjects including mathematical understanding. The reporting is also positive for students nationally. Impressive increases have also been seen in SAT and ACT Math scores and Advanced Placement examination participation in terminal high school courses such as calculus and statistics.

However, while we celebrate these record high NAEP scores and increases in SAT and ACT achievement - despite a significantly larger and more diverse range of test-takers - other recent data at Girl's High demonstrate that we are not moving forward. In the PSAT and SAT, Problem Solving and Data Analysis category, Girls' High students have not been scoring well . As shown in the tables below, even after three years, scores are flat year to year.

PSAT Data Sophomores 2018



PSAT Data Juniors 2019



The NCTM Principles to Action report also indicates there are still large disparities among racial and ethnic minority groups. The report says: "...too few students – especially those from traditionally underrepresented groups – are attaining high levels of mathematics learning. In a math classroom, a focus on critical thinking and problem-solving skills greatly increase math proficiency.

My students are as diverse as the City of Philadelphia having applied and gained admission to Girls' High from diverse neighborhoods and middle schools of every type (public, private, parochial, magnet and neighborhood). Our diverse student population while representative of Philadelphia neighborhoods across the city has become increasingly dominated by Black, Indigenous and People of Color (BIPOC). These BIPOC students represent multiple cultures and ethnicities. Dr. Christopher Emdin, in his book, "For White Folks Who Teach in the Hood…and the Rest of Y'all Too, Reality Pedagogy and Urban Education", proposes that educators embrace students' cultural differences by adopting pedagogy which connects positively with their differences and "become an ally who is working to reclaim their reality".(Emdin, 2016) This unit will not unpack all of the changes needed to offer sensitive, culturally relevant environment and lessons for BIPOC students but recognized that their learning styles compared to the more Eurocentric educational structures which dominate American education and doesn't always support BIPOC student learning styles.

The acceptance the importance of learning styles and brain modalities has become common place among educators, administrators and even parents. In f act, in his article Ethnocentric Origins of the Learning Style Idea, Thomas Fallace states:

> In recent decades, the idea that teacher should align their instruction with students' particular learning style, cognitive style and/or learner preference has become commonplace in the literature on effective teaching. (Fallace,(2019).

Its popularity withstanding, many recent studies written in educational literature have questioned whether aligning teaching with student learning styles is beneficial and appropriate. (Hushmann & O'Loughlin (2019); Knoll, Orani, Skeel, & Van Horn,(2017), Pashler etal., (2009); Knoll (2017) . The notion that students were different along characteristics which benefitted from individualized instruction began after World War II and intensified in the 1960's during what is called the progressive movement (Cremin, 1961; Zilversmit, 1993). While the research focus acknowledged both different learning styles and intelligence, later the focus changed eliminating learning difference considerations and related intelligence and achievement fueling conclusions that learning differences among ethnic populations were an indication of intellectual inferiority based on race. (Fallace, 2019) The arguments regarding biological justifications of inferiority driven by racist ideas and the cultural relativistic stance which held that social inequalities, unjust social policies, cultural differences, and biased assessments explained the achievement differences began to gain acceptance in the 1940's and continued throughout the 1960's (Fallace, 2019). Eventually while some research eluded to a Black learning style most research avoided the distinction regarding race possibly to avoid the controversy related to learning styles and racial superiority compared to the dominant culture. The research of Dunn, etal (1975)and Kolb (1976) established what the educational community considers the standard learning styles philosophy but makes no mention of learning styles and ethnic or racial groups. (Fallace, 2019).

Some African American scholars continue to pursue the idea that Black students may have a learning style that is significantly different enough to effect learning particularly when pedagogy is designed primarily for White students Hale, (198)2, Shade (1982)

There are seven learning styles which educational literature defines relate to student learning.

The Seven Learning Styles

Visual (spatial):You prefer using pictures, images, and spatial understanding.

Aural (auditory-musical): You prefer using sound and music.

Verbal (linguistic): You prefer using words, both in speech and writing.

Physical (kinesthetic): You prefer using your body, hands, and sense of touch.

Logical (mathematical): You prefer using logic, reasoning, and systems.

Social (interpersonal): You prefer to learn in groups or with other people.

Solitary (intrapersonal): You prefer to work alone and use self-study.

Source: https://www.learning-styles-online.com/overview/

Learning style preferences are not uniform across any group of students regardless of racial group however researchers have consistently found trends useful to consider for racial groups. Pat Guild reports in the article, Learning Styles of African American Children: Instructional Implications that researchers Hale-Benson (1986), Shade(1989) and Hilliard (1989) have reported three kind of information about culture and learning styles among Black and Brown students learning math:

1. Many African American students tend to respond in tern of the whole picture instead of the parts. They would investigate the whole visual rather than studying one part at a time.
2. Many African American students tend to prefer inferential reasoning to deductive or inductive reasoning. Inference plays a vital role in constructing mathematical knowledge.
3. Many African American students tend to approximate space, number, and time rather than stick to accuracy
4. Many African American students tend to prefer novelty, freedom, and personal distinctiveness – they like variety

The lessons created in this unit will aid visual spatial learners while also reaching auditory sequential learners. How are students with these learning styles different? Auditory sequential learners prefer sequential reaching methods where they listen (auditory). They easily recall math facts, memorize steps to complete math problems answer the practice problems with ease and may not learn the concepts deeply or understand the underlying mathematical concepts without specific work to learn it. (Hass 2003). Visual spatial learners by contrast need to translate auditory inputs into visual images to learn and apply the information. They learn concept holistically rather than in parts. (Haas, 2003; Silverman, (2002) The visual spatial learner creates a picture, video, photograph, icon, or another image to. aid in the translation process. (Freed, Kloth, & Billett, 2006; Haas, 2003; Silverman, (2002). They “see” real world applications more easily than other learners and often understand complex problems more readily than simpler ones because they can start with the whole and then tackle the problem.

Auditory Sequential Learners Left Hemisphere/Left Brain Learners	Visual Spatial Learners Right Hemispheric/Right Brian Learners
• Think primarily in words • Hand a good sense of time • Are step by step learners • Follow oral directions we • Are well organized • Memorize linear instructions and arrive at one correct answer • Progress readily from easy to difficult material (Silverman, 2002)	• Think primarily in pictures • Relate well to space but no time • Are whole concept Learners • Read maps well • Have unique methods of organization • Learn best by seeing relationships or patterns • Learn complex concepts easier than simple ones • (Silverman, 2002)

Each of the lessons created for this curriculum unit have elements beneficial for both auditory sequential learners (steps) and visual spatial learners (whole problem).

Backward design is also beneficial strategy both for the teacher and the learners of different learning styles. Wiggins and McTighe Backward Design, planning with the end in mind, helps the teacher establish where she is going and then lay out the lessons to achieve the goal. In these lessons the essential question represents with the student will know when they complete the lesson. Particularly for the visual spatial learner knowing the big picture first, where the learning is going is helpful in helping them to frame the work ahead as pictures and chunks of knowledge. There are also directions step-by step for the auditory sequential learners to follow methodically as they complete the lesson.

Teaching Objectives

Students will learn

- Determining expressions and values using mathematical procedures and rules
- Connecting representations
- Justifying reasoning and solutions
- Using correct notation, language, and mathematical conventions to communicate results or solutions

My curriculum unit will provide opportunities for students to experience the math content with visual animations and to use manipulatives designed specifically for the lesson so that they have a personal experience with the content. Curriculum unit lessons will allow students to see mathematical transformations physically move across the smartboard based on the changes in mathematical functions in x-y space. These animations will bring the mathematical concepts to life visually much like the instantaneous movement they encounter in numerous video representations.

My students struggle with concepts and seeing past the numbers to higher understanding allowing them to think critically about the content. The first lesson, for Algebra 2 students, provides a visual representation of translations. Specifically, seeing the animations and then crating animations of vertical translations. I have chosen vertical translations because the “k” values can be derived directly from the quadratic equation. By comparison, the horizontal translation and the “h” variable changing in the opposite sign compared to its direction is a major stumbling block best addressed once the idea of movement and a numerical change causing that movement are firmly established. Students will make a flip book to demonstrate the “k” value changing and the graph moving. They will be able to see the graph move and ultimately relate it to the formal definition of quadratic equations.

The second and third lessons will focus on calculus with an introduction to Riemann and an accumulated area/Riemann sums word problem. My calculus students struggle to understand the approximation of area using the rectangles filling the space under the graph of the function and the x-axis. The idea that the approximate area gets better as the number of rectangles increases is understood a rote level as a process but not a transferable skill to carry them into calculus and the fundamental theorem of calculus. Students begin calculus with an in-depth study of limits. Historically as we enter the study of integrals and draw on limits and Riemann sums, students do not make the connection . It is therefore extremely difficult to successfully complete the integration word problems and real-life application problems.

Classroom Activities

Lesson Plan 1 –Transformations of Quadratic Equations: Making a Math Transformation Flip Book

Essential Question(s):

- What are the characteristics of quadratic function translations?
- How do the constant a, h, and k affect the graph of the quadratic function $g(x) = a(x-h)^2 + k$

Lesson Objective(s)

Students will demonstrate knowledge of function families

- Students will describe transformations of quadratic functions.
- Students will write transformations of quadratic functions.

Previous Learning: Students have prior knowledge of basic transformations from middle school mathematics. They were introduced to transformations in the previous chapter.

New Vocabulary: quadratic function, parabola, vertex of a parabola, vertex form

Previous Vocabulary: domain, range, transformations

Teaching Materials: PowerPoint –quadratic translations, student warm up sheet, student flipbook sheet

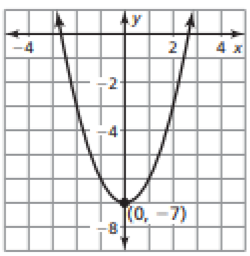
Materials for Students: graphing calculator, graph paper, supplies for making math transformation flip book.

Warm-up:

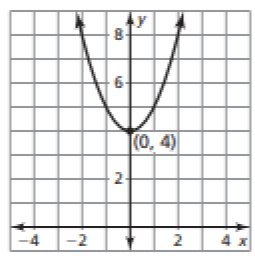
Students will use a graphing calculator and graph each of the quadratic functions given. They will match each quadratic function with its graph. They will describe their reasoning by list what they noticed the differences and similarities are for each function. (teacher supplied from textbook). Ask students to discuss their findings in groups of 2 to 4 students. When the groups report out to the larger group, encourage comments comparing the graphed problems to the parent function.

Example of problems you might select from your textbook:

1. $g(x) = x^2 - 7$



2. $g(x) = x^2 + 4$



Introduce New Vocabulary

The following vocabulary will be new to most students: quadratic function, parabola, vertex of a parabola, vertex form. Define and provide visual representations of each word. Most algebra 2 textbooks and the internet have descriptions and images for each word

Core Concept: Horizontal and Vertical Translations

Referring to the warm-up graphs discuss the form of the parent function equation f(x) = x^2 with one of the problems. Example $g(x) = x^2 - 7$.

Introduce the general form of the quadratic equation.

Tell your students that the parent function of a quadratic equation f(x) = x^2 has that form because it has its vertex on the origin (0,0).

The general equation of a quadratic function is:

$$g(x) = a(x - h)^2 + k$$

Comparing equations define a, h, and k spending time on the k values for the parent function and those of the problems from the warm-up.

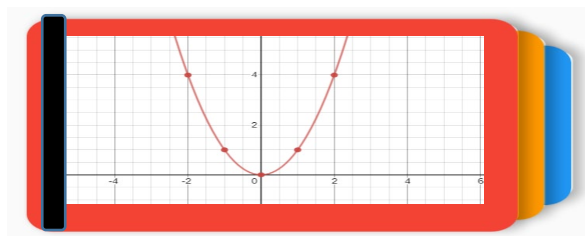
Students will make flipbook for vertical translations.

What is a flipbook? A flipbook is a small stack of index cards where each card contains a pictures or image. When you flip through cards an animation appears of the pictures or drawings.

Students will use a flip book to animate translations of quadratic equation graphs. Working in pairs students will use their cellphones to film each other's animations.

Supplies:

- Stack of index cards (pack of 50 cards is a good number to start)
- Rubber band or a binder clip with rigid handles (bull clip)
- Scissors
- Glue stick
- Paperclip (optional)
- Copies – Student flip book sheet (one graph per index card)



Flip Book Example

Instructions: (make a copy for each student)

1. Cut the parent function graph from the student flip book sheet (leave the other blank graphs together on one sheet). Cut the parent graph slightly smaller than an index card. In the top left corner write a small #1
2. Place the parent function graph beneath the first blank graph sheet.
3. Position the vertex of the parent function parabola a tiny bit up (maybe 1 tick-mark up) from the origin of the blank graph. This graph will be the first movement in your animation, so it should represent a small shift from the first graph.
4. Trace the parent graph on the blank graph with a pencil so you can erase if you make mistakes (you can make the marks darker before you make you photocopies). Paperclipping the two-graph sheet together before you write may make tracing easier.
5. In the top left corner write a small #2 on the new graph
6. Place the parent function graph beneath a new blank graph sheet
7. Position the vertex of the parent function parabola a tiny bit up (maybe 2 tick-marks up) from the origin of the blank graph. This graph will be the second

movement in your animation, so it should represent a small shift from the previous graph.

8. Trace the parent graph on the blank graph
9. In the top left corner write a small #3 on the new graph
10. Continue tracing using the parent function graph and a blank graph page until you have graph sheets going close to the top of the graph sheet. Each time move the parent graph up a small amount. Each graph will represent another movement in your animation, so make certain to move a small shift from the previous graph.
11. Each time you complete a new graph, write the next consecutive number in top left corner of the graph
12. When you finish tracing the last graph, go over the pencil lines of each graph with the same dark color ink or marker so they will make legible photocopies
13. Use a photo copier and make 3-4 copies of your graph sheets (if you do not have a photo copier, you will need to trace duplicate copies of each graph)
14. Cut all the graph sheets of your drawing apart from each other (including the photocopies)
15. Cut each graph so that they are a little smaller than the index card
16. Using a glue stick, paste a copy of each graph sheet on a single index card
17. Place the graph cards in numerical order, smallest number on the bottom
18. Make a stack of graph cards placing the cards in numerical order starting with the lowest number card on the bottom.
19. You may wish to add a top and bottom index card for a front and back cover. Feel free to decorate these cards with your name and nice illustrations
20. Secure left edge of the pages with a rubber band or a binder clip (bull clip).
21. Flip through your book starting with the bottom page as if you were shuffling a deck of cards. The pictures of your graph tracings should start at the x axis and move up the y axis.
22. Ask a friend for help. Take out your cellphone and film your animation while your classmate flips the pages of our flipbook.

Congratulations, you have created a vertical translation of the quadratic parent function $y = x^{2!}$

Lesson Plan 2 - Part 1 Accumulation Function and Riemann Sums

Essential Question(s):

- How can we use the measure of area under a curve to discuss net change of a function over time?
- How are the properties of accumulated sums related to the Riemann sum definition?

- How is the area under the curve and the definite integral related?

Lesson Objectives:

- Students will understand the definition of Riemann sums.
- Students will understand the definition of Riemann sums.

Previous Learning:

Student will have experience graphing functions, finding area of geometric figures and function notation.

Previous Vocabulary: General derivative, graph functions, area, triangle, rectangle, limits.

New Vocabulary: Accumulation function, Riemann sum,

Teaching Materials:

PowerPoint presentation "Introduction to Riemann Sums,
Student materials (see list below and appendix),
Student materials:

Student Handout 1 – Triangle (4 copies)
Coin sheets (1 each half dollars, quarter, nickels, pennies)
Student Worksheet
Glue Stick
Scissors

Warm up:

Textbook (select problems requiring graphing and interpreting graphs

Core Lesson:

Introduce the accumulation function

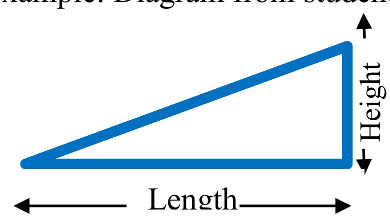
Students should think of the accumulation function as an "area over an interval" of a function. For some input x, the value of F(x) is the area under the function from a point "a" to x. Use the PowerPoint presentation to demonstrate the accumulatio of area in a triangle using coins.

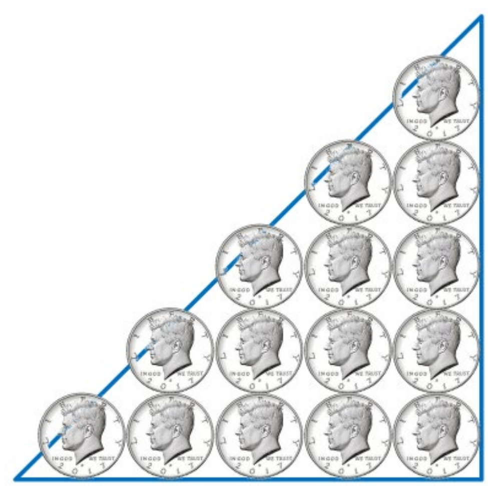
__Lesson Activity Instructions__

Using the instructions below, guide student work finding the area of the triangle and then the area of each triangle using the four coins.

<table>
<tr><td colspan="2">Teacher Guidelines Using Student wroksheet</td></tr>
<tr><td>Using the triangle given, measure the length and width and calculate the area of the triangle using area formula.

½ length x height = Area

½ * _____ * _____ = _________</td><td>Example: Diagram from student worksheet
</td></tr>
<tr><td colspan="2">For each of the four coins, half dollars, quarters, nickels, and pennies, cut out the coins from the coin sheet. Arrange the coins one at a time in separate triangles in rows and columns as shown to the right below. Calculate the area of the triangle represented by the coins using the left column</td></tr>
<tr><td>Coin: Half Dollar

Number of Coins _______________
Diameter of Coin _______________
Radius of Coin _________________
Area of Coin ___________________
Triangle Area Using Coin:
(Coin Area) x (Number of Coins) =
____
Approximate coin area outside triangle =
____
Final Triangle area: _______________</td><td></td></tr>
</table>

(see student handout for further instructions using additional coins)

Writing in Math/Discussion

Ask student to use their own words to compare and contrast the triangle areas found for each coin. What might they conclude about the effect of changing the size of the coin on the area of the triangle? What effect does each coin have on the area around the coin in each triangle? What general conclusion can be reached about smaller coins and the accuracy of the area estimation? What prior calculus concept is demonstrated here? (guide to discussion to apply limits to this problem)

Lesson Plan 2 – Part 2 Word Problems Using Riemann Sums

PowerPoint to introduce the word problem

Essential Question(s):

- How can we use the measure of area under a curve to discuss net change of a function over time?
- How are the properties of accumulated sums related to the Riemann sum definition?
- How is the area under the curve and the definite integral related?

Lesson Objectives:

- Students will understand the definition of velocity and accumulation function.
- Students will understand the definition of Riemann sums demonstrated in a word problem.

Previous Learning:

Student will have experience graphing functions, finding area of geometric figures and function notation.

Previous Vocabulary: General derivative, graph functions, area, triangle, rectangle, limits, accumulation function, Riemann Sums.

New Vocabulary: Partitions

Teaching Materials:

PowerPoint presentation "Velocity Word Problem and Riemann Sums"
Word Problem

A particle starts at $x = 0$ and moves along the x-axis with velocity

$v(t) = t^2$ for time ≥ 0. Where is the particle at $t = 3$?

Graph the function described in the word problem. Have students explain the significance of the directions.

Graph v (t) and divide the area under the curve from t = 0 to t = 3 into rectangles of equal width or partitions, representing the time subintervals used of length Δt . If $\Delta t = 1/4$, you will use 12 subintervals for the problem.

Students should find the area from t= 0 to t=12 for 12 partitions. The area for each partition is given in the graph. The area for increased numbers of partitions investigate how the area approximation changes with increasing numbers of partitions.

Student Work

Students will complete repeat the problem using 24 and 36 partitions and compare the areas of each calculation.

Resources

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Teacher Resources

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Luigi Proserpio and Dennis A. Gioia. "Teaching the Virtual Generation" *Academy of Management Learning & Education*, vol. 6, No. 1, 2007, pp. 69-80

Visual Math Improves Math Performance. https://www.youcubed.org/resources/visual-math-improves-math-performance/#:~:text=Pictures%20help%20students%20see%20mathematical,see%20the%20creativity%20in%20mathematics.

Students Resources

The Math Leaning Center

https://www.mathlearningcenter.org/resources/lessons/visual-mathematics

Mathematics courses for grades 5-10. These pdf format math courses aid in core math subject and visual math skills

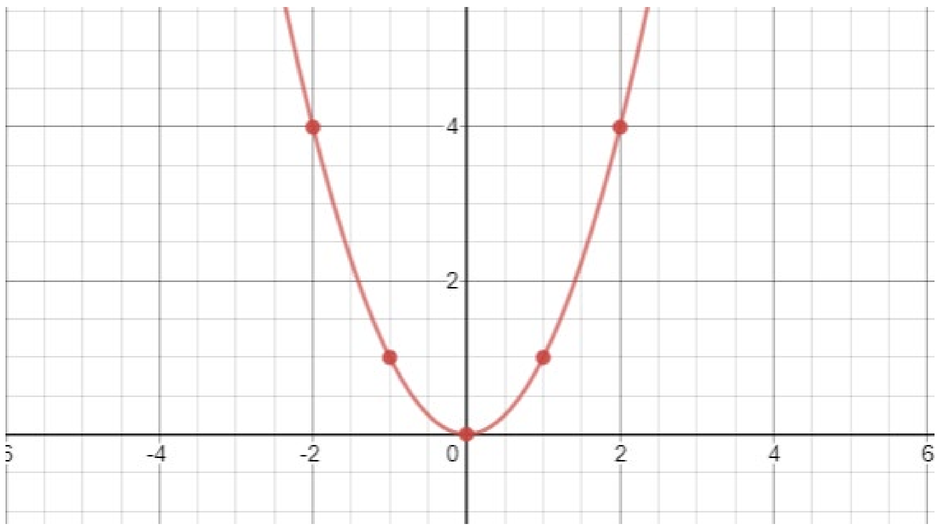
Khan Academy Left and Right Riemann Sums

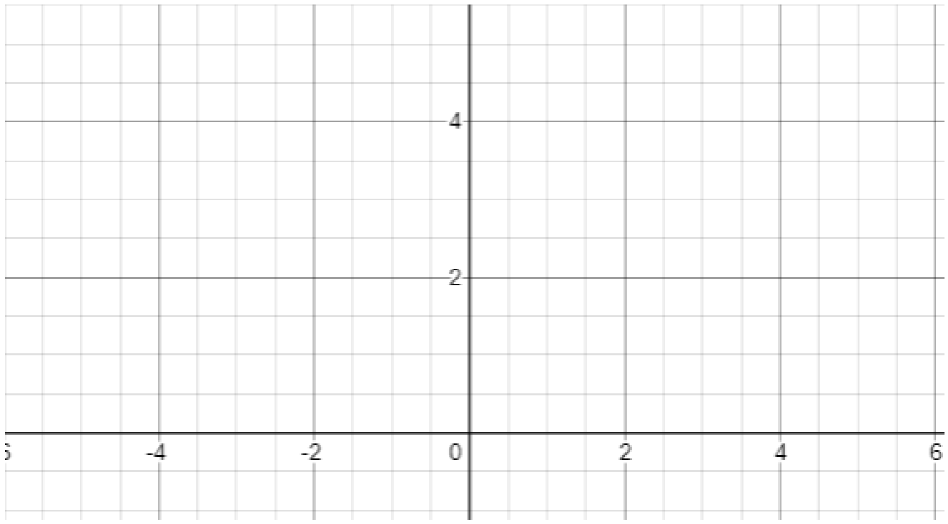
https://www.khanacademy.org/math/ap-calculus-ab/ab-integration-new/ab-6-2/a/left-and-right-riemann-sums

Appendix

Lesson 1 Student Materials

Student Flipbook Sheet

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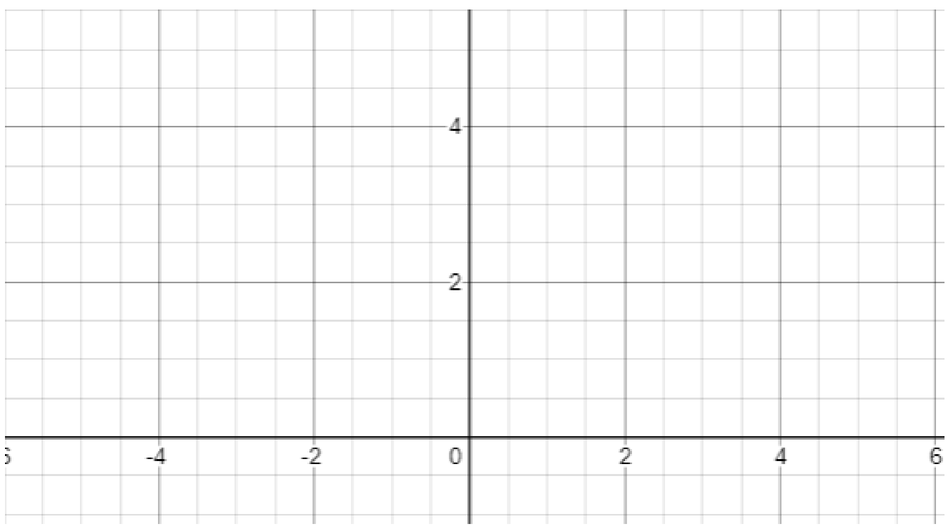
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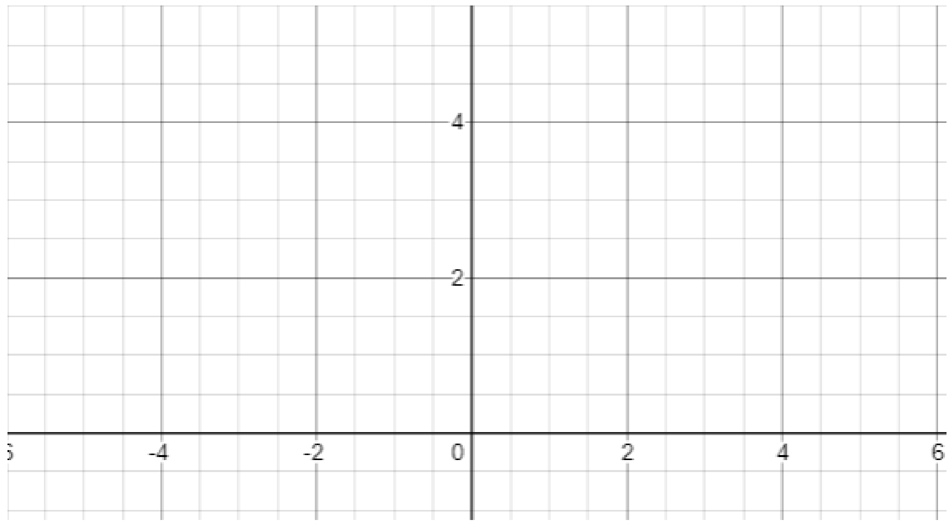

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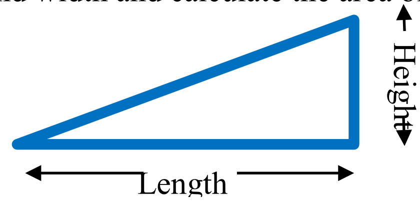
Lesson 2 Part 1 - Student Worksheet Instructions

Using the triangle given, measure the length and width and calculate the area of the triangle using area formula.

½   length x height = Area

½ * _____ * _____ = _________



For each of the four coins, half dollars, quarters, nickels, and dimes, cut out the coins from the coin sheet. Arrange the coins one at a time in separate triangles in rows and columns as shown below. Calculate the area of the triangle represented by the coins using lines in the left column for each coin.

Coin: Half Dollar

Number of Coins _______________

Diameter of Coin _______________

Radius of Coin _________________

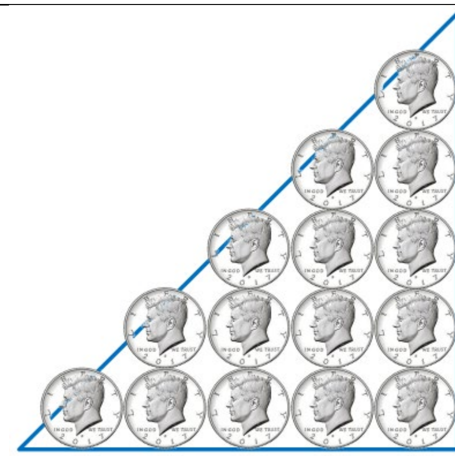
Area of Coin ___________________

Triangle Area Using Coin:

(Coin Area) x (Number of Coins) = _______

Approx. coin area outside triangle = _______

Final Triangle area: _______________



Coin :Quarter

Number of Coins _______________

Diameter of Coin _______________

Radius of Coin _________________

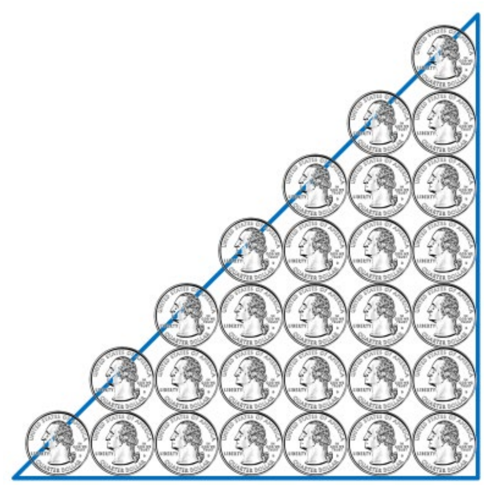
Area of Coin ___________________

Triangle Area Using Coin:

(Coin Area) x (Number of Coins) = _______

Approx. coin area outside triangle = _______

Final Triangle area: ______________



Student Worksheet Instructions

Coin: Nickel

Number of Coins _______________

Diameter of Coin _______________

Radius of Coin _________________

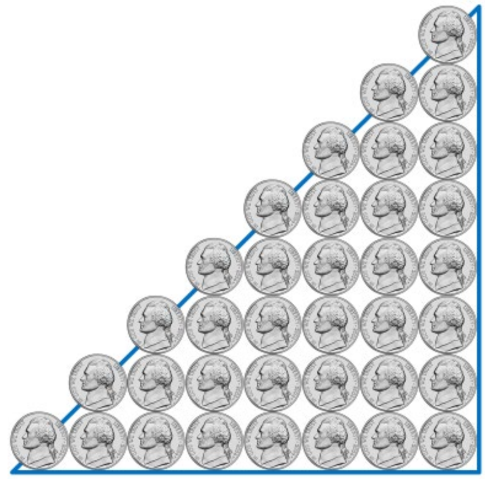
Area of Coin ___________________

Triangle Area Using Coin:

(Coin Area) x (Number of Coins) = _______

Approx. coin area outside triangle = _______

Final Triangle area: _______________



Coin :Dime

Number of Coins _______________

Diameter of Coin _______________

Radius of Coin _________________

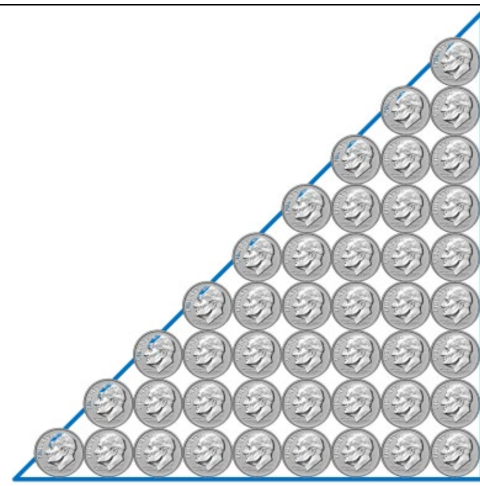
Area of Coin ___________________

Triangle Area Using Coin:

(Coin Area) x (Number of Coins) = _______

Approx. coin area outside triangle = _______

Final Triangle area: _______________

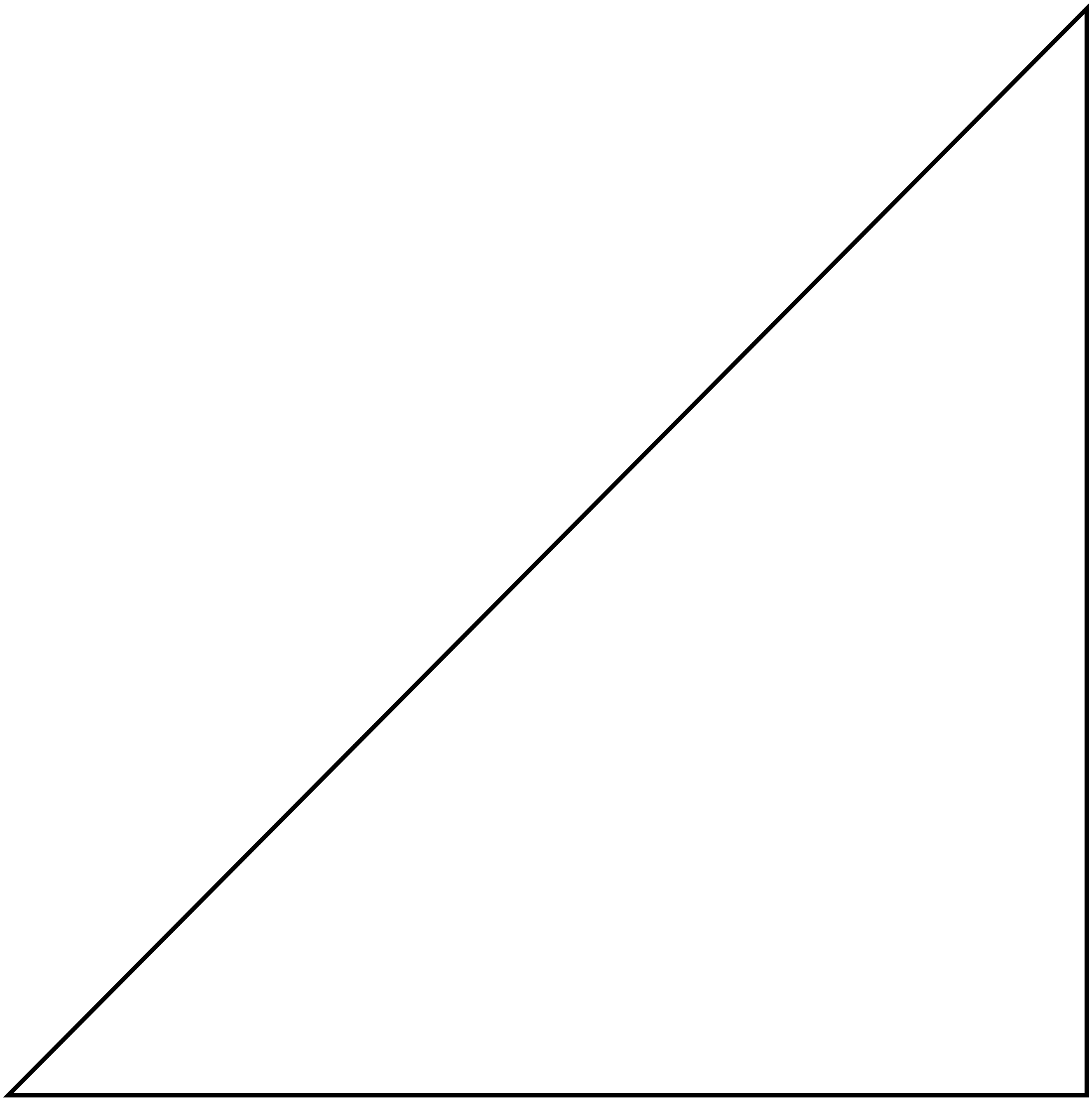


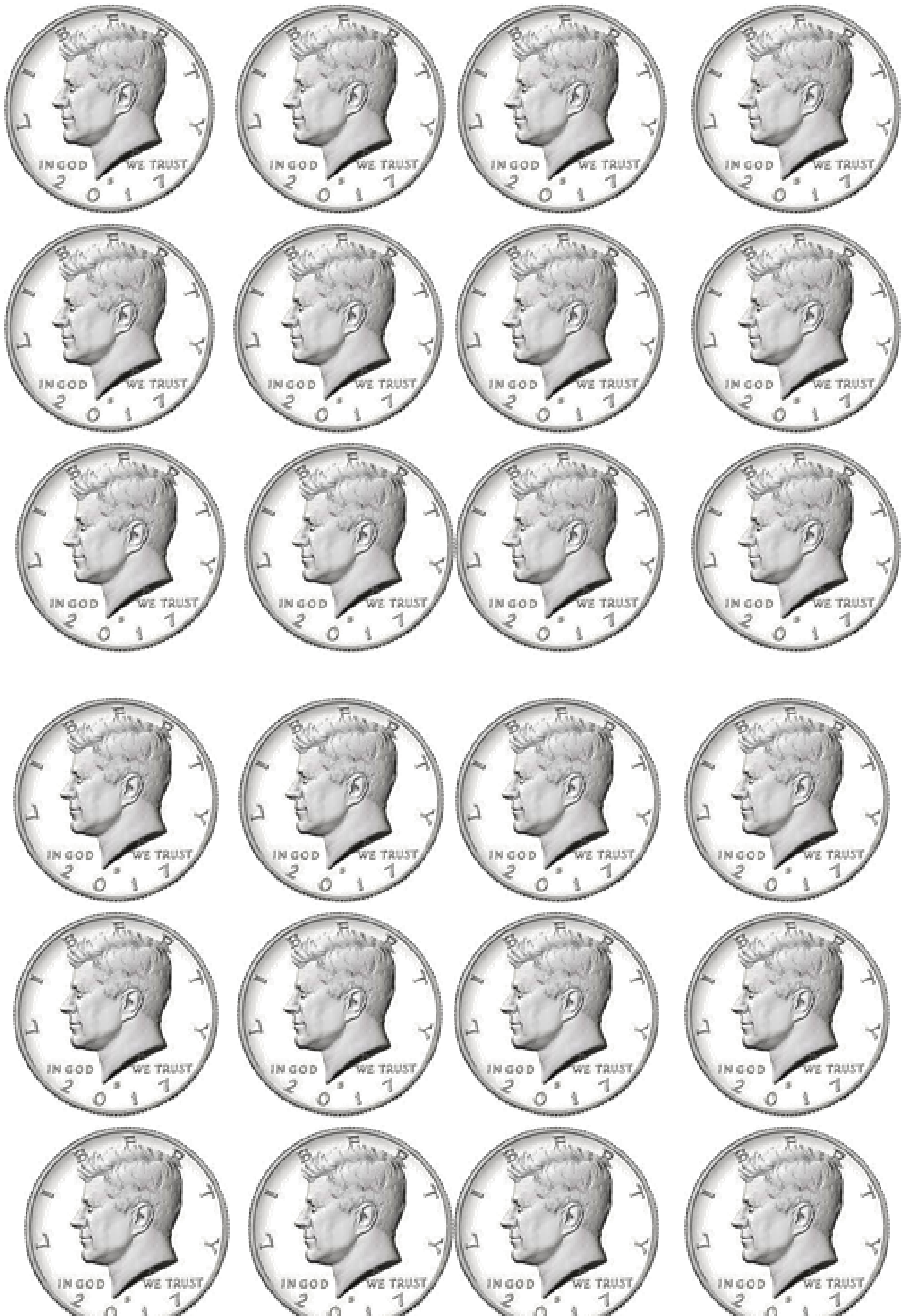
Discussion:

Notes:

Conclusion:

Student Triangle

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IN GOD WE TRUST
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UNITED STATES OF AMERICA
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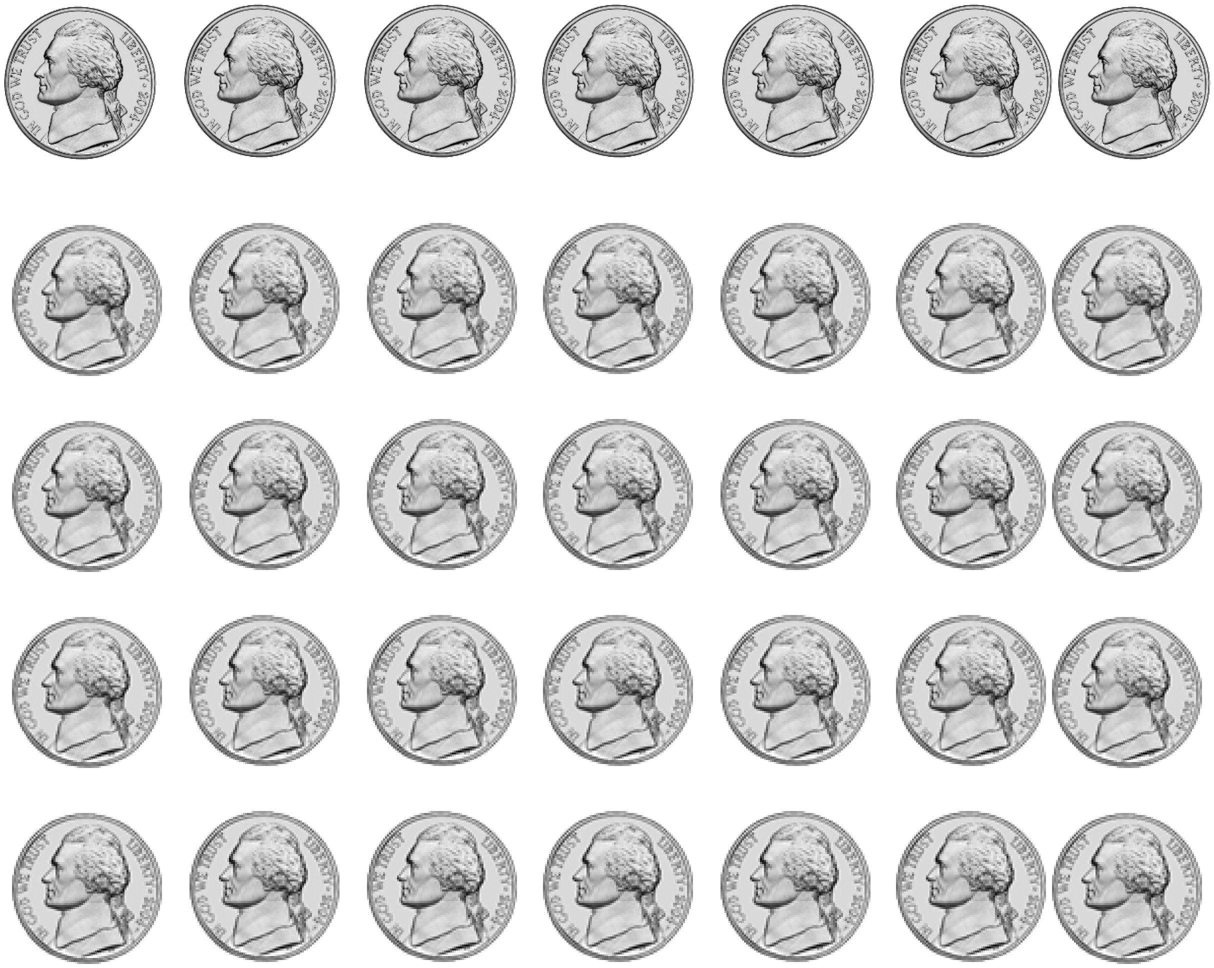

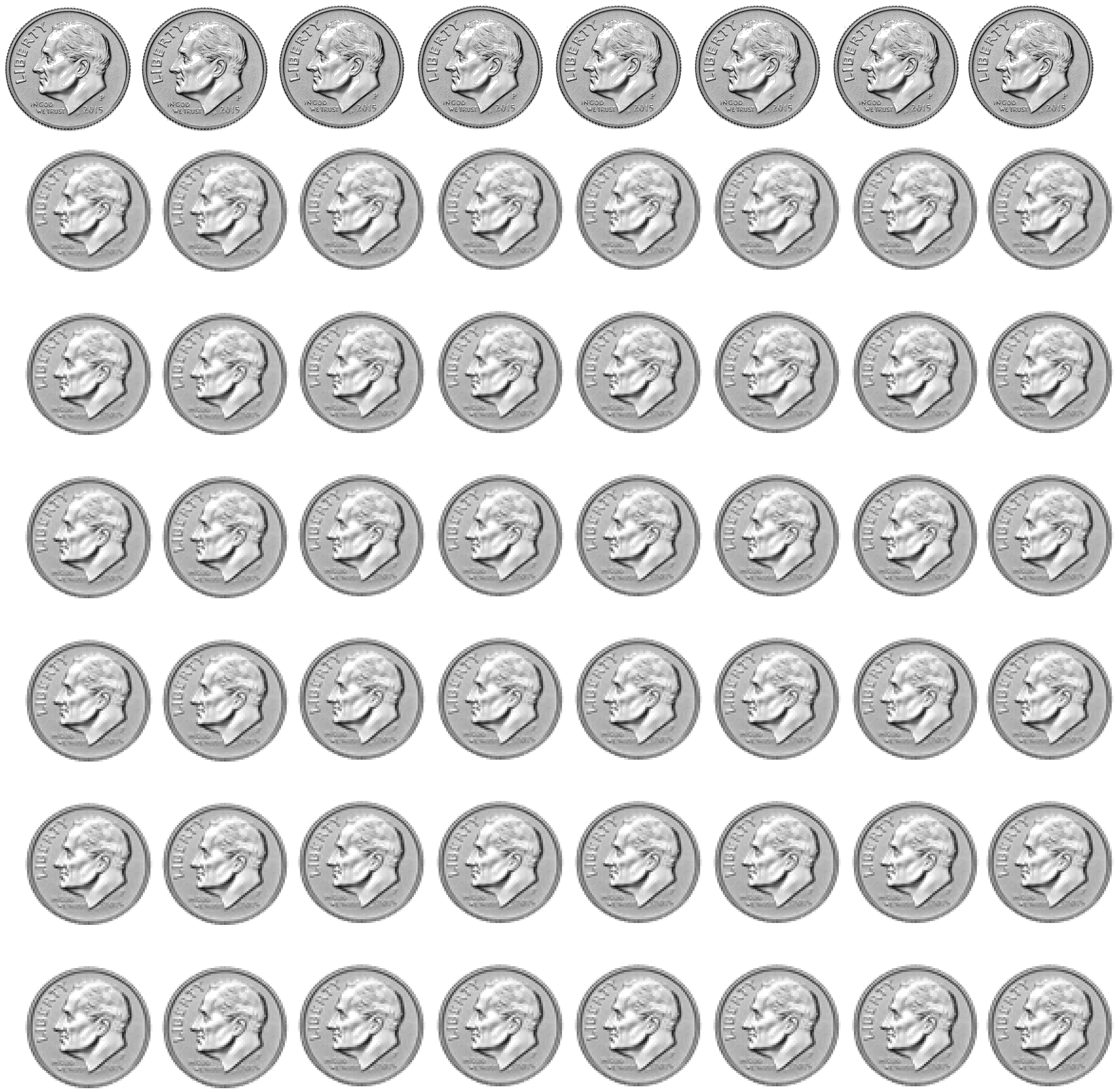

Academic Standards

Calculus Common Core Standards
C.I.UI.A.3 Use Riemann sums (left, right, and midpoint) and trapezoidal sums to approximate definite integrals of functions, represented graphically, numerically, and by tables of values. C.I.UI.A.1 Define the definite integral as the limit of Riemann sums and as the net accumulation of change. C.I.1: Use rectangle approximations to find approximate values of area. C.I.3: Interpret a definite integral as a limit of Riemann Sums.

Algebra PA Common Core Standards
CC.2.2.HS.C.3 Write functions or sequences that model relationships between two quantities. CC.2.2.HS.C.4 Interpret the effects transformations have on functions and find the inverses of functions. CC.2.2.HS.C.5 Construct and compare linear, quadratic, and exponential models to solve problems. CC.2.2.HS.C.6 Interpret functions in terms of the situations they model. CC.2.2.HS.D.7 Create and graph equations or inequalities to describe numbers or relationships.