# Visualizing Fractions

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## Content Objectives

*Penn Alexander School* is located at 4209 Spruce Street. It is a K-8 school with a population of 566 students. Of that population 60% of students are students of color representing China, Bangladesh, Sudan, Bengali, Mexico, Egypt and Russia. This unit is being written for a third- grade class, age range 8-9 years.

Students are assessed in the Spring of each school year by the Pennsylvania State Department of Education. In the Spring of the 2018-2019 school term, 68 third graders took the Pennsylvania State Standardized Assessment in Math. For the standard Numbers and Operations- Fractions the average score was a 4 out 7 points. The lowest average across all math standards assessed. Additionally, for the 2017-2018 school term the average score was 3.8 out of 8 possible points. This data shows that overall, third grade students struggle with the concept of fractions. Although many factors can contribute to these scores, the two I think deserve a closer look are - the visual representations the curriculum includes and students' conceptual understanding of fractions.

During the Teacher's Institute of Philadelphia at the University of Pennsylvania, in the Spring of 2020, I was able to take the class: *A Visual Approach to Learning Math* with Professor Robert Ghrist. Throughout the course we viewed, created and learned about many examples of graphic design. The focus was mainly on design, illustration and animation using Power Point. In addition, we were provided examples of 'good and bad' mathematical visuals and given tips and techniques for creating our projects. Before taking this course, I knew the value of visuals in teaching of math, specifically fractions. However, I became eager to learn how to implement Power Point presentations into my current teaching practice in order to supplement the lessons I teach around fractions and beyond.

Before I could begin creating student-friendly Power Point presentations, there were a few design tips to consider. To start, some of the most important design features are: layout, font and color. All of these features can influence the viewers experience. Font and color are what make the first impression. Today's students have limited attention spans and using the correct design elements can grab their attention and keep them engaged.

Specific colors can provoke associations and feelings. Blue, green, purple and red are the top color choices for Americans. Research shows that using a neutral color for the background and a dark color for the text is a good combination. Colored text on a white background or vice versa.

In addition, fonts should be easy to read and kid-friendly. The easier the text is to read; the more students will enjoy the presentation which in turn will increase their engagement. There are plenty of child-friendly fonts, comic book font was one suggested during this course.

Depending on the type of presentation, layout is another feature that is important. Layout refers to the arrangement of the images and text on a slide. The simpler the layout the easier it will be for students to understand what it is you're trying to teach. Students will respond better to graphics and photos over just text. The visuals will spark more interest and engagement.

Another design feature I wanted to include in this unit is animation. Animation is a special effect added to text or an image in a presentation. I chose to use morphing as I found this to be the animation my students and I could easily learn and utilize for creating presentations. Morphing allows objects or shapes to move and this will be valuable to use when pulling out equal parts of a whole in a fraction presentation.

This class also inspired me to question further the benefits of visual learning in mathematics. When students read a story, they learn to picture the events in their minds, which in turn helps them boost comprehension and understanding of the setting, characters, plot, and more. Likewise, in math, students can understand a concept better if they can form a mental picture of the idea being studied.

All children are visual learners. From the time we are born we are trying to make meaning of visual information around us. Young children are able to communicate with each other by creating pictures or images. They learn that lines, shapes and colors relay meaning. Studies have shown that children can make connections to different types of learning by creating images. All teachers understand that there are many types of learners. Differentiated instruction helps to meet the needs of all of the students in our classroom. In regards to visual learners, we incorporate the use of concept maps, diagrams, charts and other images to allow for visual learners to be involved fully in the learning process.

Research has shown that visual learning is appropriate in meeting those wide range of learners. Visuals help them to understand abstract math concepts by helping them to see how they work. Math teachers today equip their students with visual models to help them solve math problems. This helps them to visualize a problem as a set of images.

Visual math learning is about obtaining information through illustrations, photos, diagrams, graphs, symbols and other visual representations. As teachers, we have to be careful in the visuals we provide. Teachers have to carefully create images or models that students can easily understand and will be able to use consistently over time. It's like a visual language. Presenting information verbally, visually and numerically helps children to fully understand mathematics concepts. In teaching math, numbers, words and pictures should come together to clearly demonstrate what is taking place. In fact, recent studies have shown that the brain uses multiple regions while students complete math, including visual and sensory motor pathways. Therefore, math for visual learners should include concrete representations that kids can see to help in their understanding of the underlying math concept.

For example, teaching 'half' in a fractions unit with just words is going to be difficult. But, if you show half of a pizza or two equal parts of a whole shape, the meaning of half becomes clear. Once half is understood, it should be easier to grasp learning, fourths, thirds and sixths. Images can express things in ways numbers and words cannot.

**Teaching Strategies**

Fractions are central to elementary education. Their importance is reflected in their emphasis in the US Common Core Standards. By the beginning of third grade, students have developed an understanding of what fractions are and how they can be used to name quantities. Students have worked on problems about equal shares of a single object or a set of objects, they worked with halves, thirds and fourths. Students are familiar with the notation and words for fractions. They learned that fractions represent part of a whole and that the part it represents is one or more equal parts, which together make the whole. They also had experience in measurement to the nearest half inch. So, by third grade students have a basic understanding of fractions.

Students in first through third grade at *Penn Alexander* use P*earson Investigations 3 Math Program*. It is an inquiry-based math program where students actively explore math concepts to develop understanding and fluency. They work collaboratively with other students, investigate and participate in problem-based learning. Unit 6 is titled Fair Shares and Fractions on Number Lines. The unit focuses on the conceptual understanding of fractions as numbers and as equal parts of a whole. Students focus on reasoning about equivalent fractions; comparing fractions; and using notation to model fractions and fraction relationships. Students build their knowledge of fractions and equivalencies as they represent and compare fractional quantities, including fractions greater than one with

pattern blocks and a number line. PowerPoint will be used to supplement this unit in two ways: teacher created presentations and student created mini presentations.

**Classroom Activities**

Research shows the predominant visual model used for fractions is a circle divided into parts. Mathematicians know that fractions aren't just pizzas or pies. If we only think of fractions as circles or parts of a circle, then transitioning students to use a number line visual for fractions will be lost. Teachers need to provide multiple representations of fractions when teaching so that students can apply fractions in multiple situations, such as sharing, a point on a line/measurement, part of a set/group and an amount/quantity. Using multiple visual and tactile representations of fractions will help students with fraction number sense. Additionally, students need to be able to provide visual representations of their own to add to their fraction number sense.

In order to prepare for the unit, I created visual representations of fractions relating to ½, part-whole and the number line using PowerPoint. In order to teach my students how to create representations, I practiced creating many PowerPoint slides and became knowledgeable on the different techniques I learned from *A Visual Approach to Learning Math* class. Next, I plan to model how to use PowerPoint and create student-friendly directions for using it to create halves so the students are able to create these visuals independently. The following lessons are meant to be supplemental to your mathematical instruction. The provided student resources can be used as mentor texts relating to fractions or as reading material to display during a unit in fractions.

This unit will include grade-appropriate activities to develop fractional proficiency in conceptual understanding. The following teaching strategies will be used:

- *Whole Group Direct Instruction*- The teacher will conduct mini lessons for the whole class using the SMART board or white board.
- *Small Group Instruction* – The teacher meets with small groups of students for reteaching and/or providing differentiated instruction.
- *Shared and Independent Work* – Students will have opportunities to work with a partner or small group and independently.
- *Hands-On Learning*-Students will work with models of fractions and pattern blocks.
- *Digital Learning*-Students will utilize technology to perform tasks or create presentations.
- *Student Presentations*- Students have the opportunity to share completed projects and receive questions and comments from peers.

The Core Curriculum of the School District of Philadelphia is aligned to the Pennsylvania Academic Standards for Mathematics. The key standards that will be included in this unit are:

**M03.A-F** Numbers and Operations—Fractions

**M03.A-F.1.1** Develop and apply number theory concepts to compare quantities and magnitudes of fractions and whole numbers.

> **M03.A-F.1.1.1** Demonstrate that when a whole or set is partitioned into y equal parts, the fraction 1/y represents 1 part of the whole and/or the fraction x/y represents x equal parts of the whole (limit denominators to 2, 3, 4, 6, and 8; limit numerators to whole numbers less than the denominator; and no simplification necessary).
>
> **M03.A-F.1.1.2** Represent fractions on a number line (limit denominators to 2, 3, 4, 6, and 8; limit numerators to whole numbers less than the denominator; and no simplification necessary).
>
> **M03.A-F.1.1.3** Recognize and generate simple equivalent fractions (limit the denominators to 1, 2, 3, 4, 6, and 8 and limit numerators to whole numbers less than the denominator). Example 1: 1/2 = 2/4 Example 2: 4/6 = ⅔
>
> **M03.A-F.1.1.4** Express whole numbers as fractions, and/or generate fractions that are equivalent to whole numbers (limit denominators to 1, 2, 3, 4, 6, and 8). Example 1: Express 3 in the form 3 = 3/1. Example 2: Recognize that 6/1 = 6.
>
> **M03.A-F.1.1.5** Compare two fractions with the same denominator (limit denominators to 1, 2, 3, 4, 6, and 8), using the symbols >, =, or <, and/or justify the conclusions.

**CC.2.1.3.C.1** Explore and develop an understanding of fractions as numbers.

**M03.C-G** Geometry

**M03.C-G.1.1** Analyze characteristics of polygons.

**M03.C-G.1.1.3** Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. Example 1: Partition a shape into 4 parts with equal areas. Example 2: Describe the area of each of 8 equal parts as 1/8 of the area of the shape.

**CC.2.3.3.A.1** Identify, compare, and classify shapes and their attributes.

**CC.2.3.3.A.2** Use the understanding of fractions to partition shapes into parts with equal areas and express the area of each part as a unit fraction of the whole.

**Lessons:**

**1. The Many Shapes of Halves**

**Standard:**
M03.A-F.1.1.1 Demonstrate that when a whole or set is partitioned into y equal parts, the fraction 1/y represents 1 part of the whole and/or the fraction x/y represents x equal parts of the whole (limit denominators to 2, 3, 4, 6, and 8; limit numerators to whole numbers less than the denominator; and no simplification necessary).

**Objectives:**

1. Students will develop an understanding of ½ and whole by trying to locate ½ in a complex geometric image.
2. Students will be able to design their own halves using squares.

**Materials:**

- "I Spy ½" sheet -Visualizing and Investigating Big Ideas, Grade 3 pg. 142 (Teacher Resources)
- Dot Paper
- Markers or Colored Pencils
- Pre- cut Square paper

**Instructional Activities:**

Visualizing Activity: Teacher projects black and white art titled "I Spy 1/2". Partners are asked where they see halves. They are given time to "turn and talk" to share observations of half. The teacher asks a pair to share where they see half. The teacher demonstrates

how to show their thinking on the art by drawing around the whole with a colored marker.

Teacher can share on overhead projector, document camera or display student work. Students will see half by talking about what it is half of. (whole)

Students are given a copy of the "I Spy ½" sheet (see Resources, Visualizing and Investigating Big Ideas, Grade 3 pg. 142) and Dot Paper *(See Appendix)*. They are given time to explore more places they see half. The students mark their sheets as the teacher modeled and then draw those shapes on dot paper showing the halves to make their thinking clear. (see Resources, Visualizing and Investigating Big Ideas, Grade 3- pg. 138) Students come together at the end of exploring and share their discoveries. Teacher can share on overhead projector, dot cam or display student work. The teacher can prompt students by asking: Where do you see half? How do you know this is half? Students will be able to see the relationship between whole and part. A half could be 1 of 2 equal parts or 3 of 6 equal parts or a triangle out of square.

Students can make real-life applications as the teacher can ask students where else they can see ½ in the classroom and what it is ½ of. Example: half can be seen in the top and bottom pane of a window or half of the students in the class are boys.

Making Halves Activity: Teacher hands out square cut paper to each student. Students color half of the square. Students switch squares with a partner to see if they agree with their half. As the class come back together, the teacher displays the different ways a square can be halved using student work. The students could do a gallery walk where they walk around the classroom viewing the different ways.

**Digital follow up**: Partners create one half visuals using PowerPoint showing half with the morphing feature. Teacher models and provides written instructions for the students to follow. (*See Appendix*) Students present their work to the whole class.

2. Making Fair Shares

**Standard:**

M03.A-F.1.1.1 Demonstrate that when a whole or set is partitioned into y equal parts, the fraction 1/y represents 1 part of the whole and/or the fraction x/y represents x equal parts of the whole (limit denominators to 2, 3, 4, 6, and 8; limit numerators to whole numbers less than the denominator; and no simplification necessary).

**Objectives:**

1. Students will be able to understand one of the equal parts of a whole as a unit fraction.
2. Students will be able to understand and use notation for unit fractions (1/2,1/3,1/4,1/6,1/8)

**Materials:**

- Large Brownie sheet (*see Appendix*)
- Sharing One Brownie Activity Sheet (*see Appendix)*
- Small Ziploc bag
- Scissors
- Glue or tape

**Instructional Activity:**

One Brownie to Share Activity- The teacher distributes the Large Brownie sheet, Sharing One Brownie Sheet and one small Ziploc bag to the students. Introduce the activity by explaining to students one way that we use fractions is by sharing something equally and have to split up a whole thing into equal parts. You will pretend the rectangles on this sheet are brownies. The students cut out the 8 brownies and place them in the Ziploc bag for safe keeping.

The teacher holds up one brownie or projects it using a document camera. Teacher asks: Suppose 2 people want to share this whole brownie. Think about different ways to cut the brownie so that each person gets an equal share. Use a pencil to draw a straight line as if you were going to cut the brownie with a knife. Allow students to share a few examples. The teacher displays different ways the students found. Teacher questions: Can both of these different pieces be ½? What would you call the piece of the brownie that one person gets? How do you write one half? The students should know how to write ½ from Lesson 1. Teacher writes ½. Teacher questions: Does anyone know why we write ½ this way? What do you think the number on top means? On the bottom? Guide the students to also express ½ as 1 out of 2 equal parts.

The teacher holds up another brownie or projects it using a document camera. Teacher asks: Suppose 4 people want to share this whole brownie. Teacher questions: What do you think we call those equal shares? How much would each person get? How do you write that fraction? What does the 4 tell us in the fraction? What does the 1 mean?

Partners will work together to figure out how to share the rest of their whole brownies. The students will work together to divide the rectangles into halves, fourths, eighths, thirds, and sixths. Remind students to use a pencil or fold the rectangles first before cutting. Students will glue their fractional pieces on the Sharing One Brownie Sheet and label each fractional piece. As students work the teacher observes their work. Teacher questions: Are these equal pieces? If these are equal pieces, how do we label each piece? How can you use the brownie divided into fourths to help you with eighths?

**Digital follow up:**

Teacher uses the Power Point presentation for Making Fair Shares as a visual to show a whole brownie divided up into halves, thirds, fourths and eighths, thirds and sixths. This can be presented after the lesson or during the lesson to provide extra support.

3. Fractions on a Number Line

**Standard:**
M03.A-F.1.1.2 Represent fractions on a number line (limit denominators to 2, 3, 4, 6, and 8; limit numerators to whole numbers less than the denominator; and no simplification necessary).

**Objectives:**

1. Students will be able to represent fractions on a number line.
2. Students will be able to understand a unit fraction as a number represented on a number line.

**Materials:**

- Number Line Activity sheet (*see Appendix*)

**Instructional Activity:**

Travelling Ant Activity- Teacher introduces a number line that goes from 0 to 1 to the class. Teacher explains that the number line represents one city/neighborhood block. One block is from 0 to 1. An ant is going to travel along this block, and it's standing at 0, ready to begin walking. The ant goes a half a block at a time, stopping and resting after

each half block. The teacher models the ant walking a half block. Teacher questions: What number should we put at this mark where the ant stops? What fraction of one block has the ant travelled? The teacher writes ½ on the number line. The teacher models the ant walking another half block and rests. Teacher questions: Where is the ant now? So, it's at the one on the number line. How many half blocks is that? How would you write a fraction that means two halves? The teacher writes 2/2 on the number line under the 1. The teacher goes back to the beginning of the number line. Teacher question: When the ant was waiting to start, how many half blocks had it gone? The teacher writes 0/2 on the number line under 0.

Teacher presents another number line that goes from 0 to 1. The teacher explains that one block is from 0 to 1. An ant is going to travel along this block, and it's standing at 0, ready to begin walking. This time the ant walks one third a block at a time, stopping and resting after each third block. Teacher question: Where would its first stopping point be? What should we label it? Teacher writes 1/3. Teacher explains that this is harder to mark than ½. Students should try to get as close to 1/3 and 2/3 of the way between 0 and 1, as possible. The teacher asks the students to help her label 0/3 and 3/3.

Teacher distributes the Fractions on a Number Line Activity Sheet. The students will mark the number lines for ½ and 1/3. Partners work together to mark more number lines showing an ant travelling. Point out that students may want to use the lines they marked with halves and thirds to help them with the other number lines.

**Digital follow up**: Teacher uses the Power Point presentation for Fractions on a Number Line as a visual to show students the relationship of unit fractions morphed onto a number line. The presentation can be used at the end of the lesson as a discussion point and to make connections.

## Resources

### Teacher Resources

Abdullah, Nasaurdin, Halim, Lilia & Zakaria, Effandi. (2014). VStops: A Thinking Strategy and Visual Representation Approach in Mathematical Word Problem Solving toward Enhancing STEM Literacy. *Eurasia Journal of Mathematics, Science & Technology Education*, 10(3), 165-174.

Boaler, Jo, Chen, Lang, Williams, Cathy & Cordero, Montserrat. (2018) SEEING AS UNDERSTANDING: The Importance of Visual Mathematics for our Brain and Learning. *youcubed.*

Boaler, Jo, Munson, Jen, Williams, Cathy. (2018) Visualizing and Investigating Big Ideas, Grade 3.

Cunnington, Marisol, Kantrowitz, Andrea, Harnett, Susanne & Hill-Ries, Aline. (2014). Cultivating Common Ground: Integrating standards-based visual arts, math and literacy in high- poverty urban classrooms. *Journal for Learning Through the Arts*, 10(1).

Murphy, Stuart. Visual Learning in Mathematics. (2006). *pearsonschool.com*

Neagoy, M. (2019) Unpacking fractions. *Association for Supervision and Curriculum Development.* Alexandria, VA: Alexander Street Press.

Schoevers, Eveline M., Lesema, Paul P. M. & Evelyn H. Kroesbergen. (2018). Enriching Mathematics Education with Visual Arts: Effects on Elementary School Students` Ability in Geometry and Visual Arts. *International Journal of Science and Mathematics Education.*

**Student Resources**

Dodds, Dayle Ann. *Full House: An Invitation to Fractions.* Candlewick, 2012.This book introduces the concept of fractions by hotel guests sharing a whole cake.

Emberley, Ed. *Picture Pie.* LB Kids, 2006.This book shows how to make drawings with whole circles and parts of circles. It could be used as the basis for a lesson on fractions.

McElligott, Matthew. *The Lion's Share. Bloomsbury USA Childrens, 2012.*
This book introduces halving with characters sharing a dessert.

Murphy, Stuart J. *Give Me Half!* MathStart2, 1996.This book is about two siblings sharing a pizza by splitting into halves.

Napoli, Donna Jo. *The Wishing Club: A Story About Fractions.* Henry Holt and Co., 2007. A story about children's wishes. They each get a fraction of what they wish for and combine fractions to equal the whole wish.

Pallotta, Jerry. *Apple Fractions*. Cartwheel, 2003. This book uses different types of apples to teach fractions. Elves demonstrate how to divide apples into halves, thirds, fourths and more.

Pilegard, Virginia. *The Warlord's Puzzle. Pelican Publishing, 2000.* This book will help teach the fractional relationships between pieces of a tangram to solve problems involving area.

Stamper, Judith. *Go Fractions*. Penguin Group, 2003. Fractions are taught through soccer. Many visual representations of fractions on soccer uniforms and of the oranges they eat.

## Appendix

Print copies of these resources for students to use as they complete the Fraction lessons.

**Dot Paper**

# The Many Shapes of Half, Digital Follow Up – Student Instructions

Using PowerPoint to show Halves - Student Directions

SLIDE 1

1. Start with a blank slide
2. Insert Shape ->Rectangle->resize and fill shape with a bright color
3. Copy and paste rectangle->change fill color and join the rectangles to form a large rectangle
4. Press control and mouse pad to Duplicate slide

SLIDE 2

5. Pull rectangles apart and align them
6. Insert a text box on each rectangle
7. Insert Equation and type ½ press return

**Large Brownie Sheet**

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| | |
| | |

## Sharing One Brownie Activity Sheet

Sharing One Brownie Activity Sheet

Cut up the whole brownie rectangles. Glue the equal pieces below.

1. 2 people want to share a brownie equally. Each person gets _____.

2. 4 people share a brownie equally. Each person gets _____.

3. 8 people share a brownie equally. Each person gets _____.

4. 3 people share a brownie equally. Each person gets _____.

5. 6 people share a brownie equally. Each person gets _____.

## Number Line Activity Sheet

An ant is traveling along the number line. The distance between 0 and 1 is one block.

1. The ant travels ½ block at a time. It rests at each half-block. Mark and label the points where it rests.

```
<---|-------------------------------------------------------------------------------------------|--->
    0                                                                                           1
```

2. The ant travels ¼ block at a time. It rests at each half-block. Mark and label the points where it rests.

```
<---|-------------------------------------------------------------------------------------------|--->
    0                                                                                           1
```

3. The ant travels ⅛ block at a time. It rests at each half-block. Mark and label the points where it rests.

```
<---|-------------------------------------------------------------------------------------------|--->
    0                                                                                           1
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4. The ant travels ⅓ block at a time. It rests at each half-block. Mark and label the points where it rests.

```
<---|-------------------------------------------------------------------------------------------|--->
    0                                                                                           1
```

5. The ant travels ⅙ block at a time. It rests at each half-block. Mark and label the points where it rests.

```
<---|-------------------------------------------------------------------------------------------|--->
    0                                                                                           1
```