Functional Gardens

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Problem Statement: Mathematical problem solving in the real-world continues to be a concern for mathematics educators. Algebra, Geometry, Calculus, and other math content subjects have long been taught in the same way. Teachers usually rely on the 'stand-and-deliver' lecture and heavily on the unimaginative textbook. Though some textbooks have changed in recent years, the central focus is still on paper and pencil, memorization of rules, and recurring use of postulates. The Curriculum and Evaluation Standards for the School District of Philadelphia Mathematics Instructional Model (NCTM 1989) articulates a vision for mathematics instruction to "meet students' needs to understand and be able to use mathematics in an increasingly challenging world" and "represent and analyze relationships using tables, verbal rules, equations, and graphs". The standards also urge teachers to give students the opportunity to be 'actively involved' in math through data analysis and statistics that are integrated into the curriculum. My hope is to show that these types of activities can be incorporated into an Algebra I course using perpetually basic concepts. Several of these are: look for a pattern, guest and check, make a list, make a table, draw a picture, diagram, work backwards, and try to simplify the problem. Along with addition, subtraction, multiplication, division, the order of operations, and several other rudimentary concepts, those just mentioned need to be incorporated into the art and crafts of problems solving.

Rationale: The student scores on standardized tests are low for Mathematics in the city of Philadelphia. Students are at risk of failing math at an alarming rate. Schools in our district are being managed by outside consulting organizations. Traditional methods have at best failed. Contemporary methods of teaching math have not been deemed reliable. Teachers are being held accountable. Students are not held responsible. Parents are outraged that their children are being promoted and yet can not figure the exact change on a simple transaction at a neighborhood store. Employers are disenchanted by having to spend extra money training new employees to solve old problems while new problems continue to arise. Our public school is still our largest training ground. It remains, with all of its flaws, the place where the most undeveloped minds attach themselves to confidence, esteem, respect, certainty and assurance. Teaching the art and craft of problem solving in math permitting students to use an ”Any Means Necessary-->--My

Way Works--" approach to the solution can cultivate realistic expectations for student success in Math.

Objective: My plan is to teach a unit on linear equations during the second quarter of a ninth grade Algebra I course next semester. The project will begin with the groups in one class learning the material typically covered in most algebra textbooks. I do not plan to pretest the students because this is new material for them. These groups will also be allowed to use laptop computers and complete a lesson on the computers covering linear equations. Additionally, they will be using TI-nSpire graphing calculators to explore slope and y-intercept. All of these methods are what I have typically taught over the past three years.

Another ninth grade class will be given several data collection activities as a unit of study for linear equations. My focus will begin with a whole class participation data collection activity (i.e., make a table). The class will work on the "Garden Border" problem in small sections at a time until the entire class has completed it. As a group they will record the dimensions of the garden (for example) for several different types of rectangular gardens. Students will then use a prepared activity sheet that requires them to draw a diagram of the experiment, describe the procedure, identify the independent and dependent variables, create a table of data, graph data, choose two representative points to connect and create a "line of best fit", find the slope and y-intercept of this line and describe it algebraically and verbally, then interpret the data through certain questions designed to create understanding of the purpose of the data and using the data to make predictions. This same format will be used for all subsequent activities during the unit of study. The authors of the book say "*Algebra Fundamentals* reflects the basic philosophy of the NCTM standards for learning, teaching, and assessment. Students have an opportunity to work collaboratively, to interact, and to develop communication skill." The whole idea is to "bring the real world into your algebra classroom."

I plan to require the class that does the experiments to keep a daily journal. It will include how they felt about the daily activities, a description of any specific new topic or topics they learned and a list of questions they still have. Each day the class will address any concerns from the previous day's activity. After several activities have been done by hand, I will instruct the class on how to analyze the data on the TI-84 or TI-nSpire graphing calculator. They will then be given the opportunity to use the calculator on another experiment. This class will also do the same graphing calculator activity on slope and y-intercept that the other class will do.

I will give each class the same test and compare scores. I will also give each class a survey to compare attitudes, interest and understanding of the use of the material in a real-world application. My hope is that the students in the experiment class will have grasped the basic concepts of linear equations as well if not better than the other class and be able to relate this knowledge in a very real way.

In this instructional unit students will work on skills that are directly tied to the Pennsylvania State System of Assessment. Students will have the opportunity to build their relevant content skills and develop an understanding of the connection between mathematics and communications. This unit is intended for students in ninth grade. They spend one fifty-three minute period in Algebra I class each school day. Additionally they have a 45 minute period for intensive math study.

* In a students own words - Define and explain the components of the slope-intercept form of a linear equation. –steepness, slant of the line, etc…
* Use the slope-intercept form of a linear equation.
* Draw a diagram of the explanation.
* Discuss Slope (steepness).
* Explain how the equation of the line helps develop patterns to predict future solutions to similar problems.

Background: The search for articles about my proposed topic was tedious. I have chosen to comment on a few. My goal next semester is to read and use most of these articles in my actual action paper. I have read a few articles in their entirety. What I gathered from the abstracts was the importance of using real-world applications and incorporating the use of the graphing calculator.

Since one of my goals is to show that data collection activities (i.e., make a diagram, list) can provide ways to teach the basic concepts of linear equations in a real-world setting, I tried to find articles to validate this. Mercer (1995) presents lessons that teach slope-intercept concepts of linear equations through the use of the graphing calculator. I have other articles and texts written by several different authors to support some of my ideas on the art and craft of problem solving.

Standards: The Core Curriculum of the School District of Philadelphia is aligned to the Pennsylvania Academic Standards for Mathematics. In the unit that I have chosen these standards include but are not limited to instruction on the following topics:

Linear Functions;

(2.8)

a. Determine the requested term in a pattern, given an explicit rule or table for the pattern.

b. Graph a linear function by completing tables of values for the function and plotting the points.

c. Identify the domain and the range of a set of ordered pairs.

(2.4)

a. Interpret data from a linearly related physical science phenomenon and perform a relater calculation.

(2.6)

a. Make predictions, given numeric data or a scatter-plot, graph, or stem-and-leaf plot.

Strategies: (Coultas 1995) suggests good computation skills alone will not guarantee that students will succeed. Computation is only one part of the broader problem-solving approach used to test mathematical skills. Just as in real problem solving situations, people often need to resolve more than a single problem; they may need to do more than one mathematical operation to solve some problems. The ability to read and think will be critical in dealing effectively with the problems students will be asked to solve. (Paulos 1988) very candidly points out though not dominated by martinets, early mathematics education is generally poor. Elementary schools by and large do teach the basic algorithms for multiplication and division, addition and subtraction, as well as methods for handling fractions, decimals, and percentages. Unfortunately, they do not do as effective a job in teaching when to add or subtract, when to multiply or divide, or how to convert from fractions to decimals or percentages. Seldom are mathematical problems incorporated into other schoolwork. Students fear real-world problems in part because they have not been asked to find solutions to such quantitative questions at the elementary level.

The lessons in this unit were developed for a workshop model. The workshop model begins with a teacher directed mini-lesson, which is focused on a grade level standard. Next, the students move into work time. This is the longest part of the lesson. Students may work independently or in small groups. During work time, teachers work with small groups to reinforce ideas, address misconceptions or provide enrichment activities. Teachers also use this time to conference with students, discuss work products and progress. The workshop model is research based and provides easily understandable expectations and predictable classroom routines. When developing specific strategies for this curriculum unit, I kept in mind the National Council of Teachers of Mathematics belief that proficiency in math comes from a combination of conceptual knowledge and flexibility to apply the conceptual knowledge in a variety of settings. Ninth grade students who participate in this unit will require an introduction to the idea of linear equations and functions, however with the support of a formula sheet and an explanation from their teacher they will have adequate knowledge to work on the problem. The idea of linear function may be new to the majority of students; however they should have the ability to complete the task. They will need to be flexible in their thinking to find patterns in tables, lists, and numeric data. Students, much like the rest of us, know much more math than they think they know.

I will be attempting, through the use of different approaches, to assist in learning and reinforcing the skills students need to know to solve problems. Through practice in the use of these skills, they will become more aware of their strengths and will be able to point out for themselves areas that need improvement. An important part of data-driven instruction is sharing with students their strengths and weaknesses. Information of this kind is easily accessible to any School District of Philadelphia School teacher though the SchoolNet Benchmark scores. Benchmark Exam scores can be printed so that the anchor, descriptors, eligible content and points earned over a number of different tests can be easily viewed. This data can then be shared with the student so that he or she can see their progress. Students should be permitted to choose a piece of eligible content that they have not yet shown to have mastered to date. Those who share common content difficulties will be allowed to work in similar groups

Due to the importance of group work the activities in this unit give the option of group work. Group work requires planning and organization on the part of the teacher. Expectations must be clearly defined and each student must have a job and understand the responsibilities of their given job. Another barrier to group work is deciding how to grade the work produced. A number of different approaches can be applied; each has its advantages and disadvantages. As with any assignment it is important that students understand the expectations and have seen a sample of work that meets the standard. Ideally, students can review work from previous years and see work that is below the standard, at the standard and exceeding the standard. As a teacher, I find it helpful to save samples of student work. Once students understand what is being asked of them it is important that they understand their role in the group. Work assignments with a list of job responsibilities should be given to each student. Prior to assigning the work it is important that students understand how they will be graded, individually or as a group Some teachers find it helpful to allow group members to have input in their partners' grades.

Classroom Activities

Part 1:
Translating Words Into Symbols

Translating words into symbols is thought of as modeling circumstances using an equation and variables. In the same way, algebraic equations and inequalities can represent the numeric relationship between two or more objects.

Explanation

Variables have important and diverse roles in Algebra. Often, they serve as placeholders in equations for which there are unknown quantities. In such cases, finding the specific value of the variable for which an equation is true yields the solution to the problem. Students are likely to be familiar with this from early elementary school, when they filled in the square to make a statement such as 9 + ☐ = 13 true.

One example of a problem that uses a variable in this way involves finding the width of a rectangle when the area and length are known. If the area is 36 square units and the length is 9 units, an equation for finding the width (w) of the rectangle is 9w = 36. In this situation, the variable w does not vary; it is a placeholder representing 4 units.

Another use of variables is to represent quantities that truly vary. In the area formula 36 = *lw*, the value of each variable depends on the value of the other. Consequently, as the value of one variable changes, the values of the other variables change, too. Used in this manner, variables serve several purposes that we will explore in the Role in the Curriculum section of this workshop.

Other examples:

- If a runner jogs one mile in eight minutes, the number of miles the runner covers in t minutes can be represented by the expression $\frac{1}{8}t$. Therefore, the equation $4\frac{1}{2} = \frac{1}{8}t$ could be used to determine how long it took to run $4\frac{1}{2}$ miles. More generally, if the distance run is d miles, the linear function $d = \frac{1}{8}t$ can be used.
- To determine the number of 1-foot tiles needed to construct a border around a garden that measures *l* feet by *w* feet, consider that the two lengths need *l* tiles each, the two widths need *w* tiles each, and four tiles are needed at the corners. According to Sharp (1995), students using algebra tiles “found it easy to think about algebraic manipulations when they visualized the tiles” and “the majority of students stated that the tiles added a mental imagery that made learning `easier. '”The equation could be 2*l* + 2*w* + 4. Equivalently, however, the following expressions also represent the number of tiles needed: 2(*l* + *w* + 2), (*l* + 2)(*w* + 2) - lw and 2(*l* + 1) + 2(*w* + 1).

Mathematical Definition

<u>Variable</u>**:** A symbol used to represent an unspecified member of some set. A variable is a "place holder" or a "blank" for the name of some member of the set. Any member of the set is a value of the variable and the set itself is the range of the variable. If the set has only one member, the variable is a constant. The symbols x and y in the expression $x^2 - y^2 = (x + y)(x - y)$ are variables that represent unspecified numbers in the sense that the equality is true whatever numbers may be put in the places held by x and y.
(James, Robert C. and Glenn James., 1976.)

Role in the Curriculum

Variables, expressions, and equations are important parts of the algebra curriculum. The National Council of Teachers of Mathematics (NCTM) states: Students' understanding of variable should go far beyond simply recognizing that letters can be used to stand for unknown numbers in equations ... The following equations illustrate several uses of variables encountered in [algebra]:

$$27 = 4x + 3$$
$$1 = t\left(\frac{1}{t}\right), t \neq 0$$
$$A = LW$$
$$y = 3x$$

The first equation illustrates the role of variable as "place holder:" x is simply taking the place of a specific number that can be found by solving the equation. The use of variable in denoting a generalized arithmetic pattern is shown in the second equation; it represents an identity when t takes on any real value except 0. The third equation is a formula, with A, L, and W representing the area, length, and width, respectively, of a rectangle. The third and fourth equations offer examples of variation: in the fourth equation, as x takes on different values, y also varies.

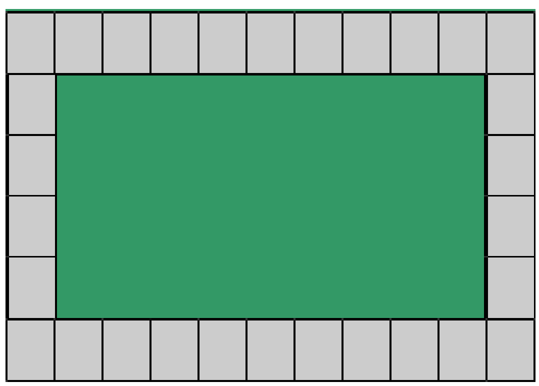
(*Principles and Standards for School Mathematics*, NCTM, 2000, p. 224)
Students should be able to relate variable expressions and equations to other forms of representations, such as tables, graphs, and verbal descriptions. One way of developing competency in this area is to use a functional approach. Present students with a table of values and have them generate a function to describe the relationship. For instance, consider the following table:

x	y
1	4
2	7
3	10
4	13
5	16

From this table, students may notice first that every time x increases by 1, y increases by 3, secondly they may notice that the y value is always equal to one more than three times the x value. Stating this relationship using variables, $y = 3x + 1$.

In a similar manner, Janel Green used a real-world problem involving the number of tiles needed to form a border around a rectangular garden. This problem provides a context in which students can use variables to represent a situation. NCTM recommends the garden-border problem as a means of developing fluency in using various representations: Students should become flexible in recognizing equivalent forms of linear equations and expressions. This flexibility can emerge as students gain experience with multiple ways of representing a contextualized problem. For example, consider the following problem, which is adapted from Ferrini-Mundy, Lappan, and Phillips (1997):

A rectangular garden is to be surrounded by a ceramic-tile border. The border will be one tile wide all around. Explain in words, with numbers or tables, visually, and with symbols the number of tiles that will be needed for gardens of various lengths and widths.



adapted from (*PSSM*, p. 282)

The example above gives students the opportunity to translate numerical situations into symbolic expressions. However, it is also important for students to translate symbolic expressions into words. For instance, students might be given the expression $6x + 3$ and asked to state a situation for which that expression could serve as a representation. A possible response might be, "Susan has three beads. She can buy more in packets of six beads each. If she buys x packets, she will have $6x + 3$ beads. " Exercises of this type could be extended to include more complex expressions and equations. In most classrooms, students typically gain extensive experience translating situations into symbolic expressions, but they generally don't encounter opportunities in the other direction. Experience with both types of situations - generating expressions for particular situations, and generating situations for particular expressions - is known as "bidirectional practice," and such practice is crucial for developing understanding.

Effectively translating words into symbols involves recognizing equivalent forms of the same relationship. Representing the same situation in more than one way provides

opportunities for students to understand equivalent algebraic expressions. The garden problem Janel Green used is one such example. NCTM states:
Complex symbolic expressions also can be examined, such as the equivalence of 4 + 2L + 2W and (L + 2)(W + 2) - LW when representing the number of unit tiles to be placed along the border of a rectangular garden with length L units and width W units. (*PSSM*, p. 225) Once students have found more than one expression that describes the number of tiles needed, they may be asked to find as many equivalent expressions as possible, and to discuss reasons why the expressions are equivalent.

In addition to meeting the NCTM Algebra Standard, the ability to interpret and describe situations in various ways helps students attain the goals of the Representation Standard. By the end of high school, students are expected to understand various representations of the same relationship and effectively represent situations using tables, graphs, and symbolic expressions. NCTM explains this further: By working on problems like the "tiled garden" problem, students gain experience in relating symbolic representations of situations and relationships to other representations, such as tables and graphs. They also see that several apparently different symbolic expressions often can be used to represent the same relationship between quantities or variables in a situation. This observation sets the stage for students to understand equivalent symbolic expressions as different symbolic forms that represent the same relationship. In the 'tiled garden' problem, for example, a class might discuss why the four expressions obtained for the total number of tiles should be equivalent. They may possibly then examine ways to demonstrate the equivalence through symbols. For example, they might observe from their sketches that adding two lengths to two widths (2L + 2W) is actually the same as adding the length and width and then doubling: 2(L + W). They possibly recognize this representation for the distributive property of multiplication over addition - a useful tool in rewriting variable expressions and solving equations. In this way, teachers may be able to develop approaches to algebraic symbol handling that are meaningful to students. (*PSSM*, p. 283) Students can solve some equations by examination or guess-and-check procedures; other equations may warrant a paper-and-pencil solution or possibly the use of technology and algebra software. A major goal of algebra is for students to acquire confidence with symbols, expressions, and equations, and to be able to represent various situations using algebraic expressions and equations. In this way, they will be able to determine which method of solution is most appropriate for a given problem.

Part 2:
Linear Equations

<u>*Linear equations*</u> can be used to model situations involving one or more unknowns. A linear equation is an algebraic equation whose variable(s) is/are of degree one. For example, 6x + 3 = 27 and a = 2b are linear equations. Solving linear equations is a key concept of the algebra curriculum.

Explanation

A linear equation is a polynomial equation of the first degree, such as $x + y = 7$. (DeTurck 2010). “Perhaps the most important concept when dealing with linear equations that relate two variables is *slope*. Indeed, the fact that lines have a well-defined slope characterizes lines (and hence linear equations) – for no other curve is it the case that for any two points on the curve, (a,b) and (c,d), the quantity (d-b)/(a-c) is independent of which two points are chosen. This is something that students might be able to discover “on their own”.”

Said another way, a linear equation has no variables raised to a power other than one. The simplest linear equations involve only one unknown, such as $x + 2 = 3$, and they are solved by finding the value of the unknown that makes the equation true. For example, the equation above is true when $x = 1$, because 1 + 2 = 3. More complex linear equations may contain more than one variable, such as $x + y = 7$ or $a - b + c - d + 4 = 14$. This workshop will focus on the simplest linear equations and those whose graph is a line.

Linear equations with just one unknown are solved using equivalence transformations, sometimes informally called "inverse operations." In this process, the operation of multiplication is "undone" using division, because the operations of multiplication and division are inverse to one another; similarly, addition is "undone" using subtraction, because addition and subtraction are inverses. The reason this process works is because the equal sign acts as a balance between the left side and the right side of the equation. As long as the same operation is applied to each side of the equation, the equation remains in balance and the equality is preserved. For example, the equation $2x + 3 = 9$ is solved as follows:

- Subtraction is the inverse of addition. Therefore, to "undo" the addition of 3, subtract 3 on both sides of the equation:

$2x + 3 - 3 = 9 - 3$, which yields $2x = 6$.

- Because division is the multiplicative inverse of multiplication, to "undo" the multiplication of x by 2, divide both sides by 2: $\frac{2x}{2} = \frac{6}{2}$, which yields $x = 3$.

Other examples:

- A traditional example involves people's ages. For instance, "Becky is 6 years younger than Sally, and Sally is 13 years old. What is Becky's age?" The equation

$b + 6 = 13$ can represent this situation, and it is true when $b = 7$, so Becky is 7 years old.

- A person bought 3 cans of soda as well as several six-packs of soda, and she has a total of 27 cans. The linear equation $6x + 3 = 27$ determines the number of six-packs she bought, where x is the number of six-packs.

- Linear functions can model the "transmission factor" of two gears. The ratio between the number of revolutions made by a 1 cm gear (b) and the number of revolutions made by a 2 cm gear (a) can be expressed as $a = 0.5b$ and $b = 2a$.

Mathematical Definition

Linear Equation or Expression: An algebraic equation or expression which is of the first degree in its variable (or variables); i.e., its highest degree term in the variable (or variables) is of the first degree. The equations $x + 2 = 0$ and $x + y + 3 = 0$ are linear. An equation or expression is linear in a certain variable if it is of the first degree in that variable. The equation $x + y^2 = 0$ is linear in x, but not in y. (Source: James, Robert C. and Glenn James, *Mathematics Dictionary* (4th edition). New York: Chapman and Hall, 1976.)

John McLeish offers an explanation of linear: "Linear means an equation of the first power of the unknown; such an equation can be represented by a straight line graph, hence 'linear.'" (McLeish, John 1991.)

Role in the Curriculum

In order to become comfortable solving and manipulating linear equations, students will need to experience linear relationships in various situations. They will need a significant amount of practice before developing fluency.

Upon the successful completion of an algebra course, students should be able to use symbolic notation to represent and explain mathematical relationships and solve linear equations. The National Council of Teacher of Mathematics (NCTM) states:
Although students will probably acquire facility with equations at different times ... students should be able to solve equations like $84 - 2x = 5x + 12$ for the unknown number ... and to recognize that equations such as $y = 3x + 10$ represent linear functions that are satisfied by many ordered pairs (x, y). (*Principles and Standards for School Mathematics*, NCTM, 2000, p. 226) Students should also be able to produce two or more equivalent expressions that represent the same situation and to use simple formulas.

Solving linear equations is a key part of attaining a global understanding of linear relationships and involves more than the ability to solve for an unknown. The solution of linear equations is often a necessary step when interpreting a complex situation. Students may describe a linear relationship using a table, graph, or words, and from that description they may generate an expression or equation to represent the situation. For instance, students may describe the cost of a cell phone plan in various ways: Some students might describe the pattern verbally: "Keep-in-Touch costs $20.00 [monthly] and then $0.10 more per minute [of use.]" Others might write an equation to represent the cost (y) in dollars in terms of the number of minutes (x), such as $y = 20.00 + 0.10x$. (*PSSM*, p. 226) From the function, students should be able to find the cost for any number of minutes. The cost for 25 minutes occurs when $x = 25$, and finding the associated cost involves solving the equation $y = 20.00 + 0.10(25)$. Similarly, the number of minutes for which the cost would be $35.00 occurs when $y = 35.00$, and students can find that value by solving the equation $35.00 = 20.00 + 0.10x$.

Lesson Plan 1: The Garden Border Problem

Overview**:**
In this lesson students will recognize patterns and represent situations using algebraic notation and variables.

Time Allotment**:**
One 45-minute class period

Subject Matter**:**
Variables, patterns, and tables

Learning Objectives**:**
Students will be able to:

- Identify a pattern involving the number of tiles required to form a border around a garden with length l and width w.
- Write a symbolic expression that describes the number of tiles needed to form a border around a garden.

Standards**:**
Principles and Standards for School Mathematics, National Council of Teachers of Mathematics (NCTM), 2000:

NCTM Algebra Standard for Grades 6-8
http://standards.nctm.org/document/chapter6/alg.htm

NCTM Algebra Standard for Grades 9-12
http://standards.nctm.org/document/chapter7/alg.htm

Procedures:

<u>Supplies</u>**:**

Teachers will need the following:

- Transparencies with 1 cm grids
- Approximately 40 unit algebra tiles

Students will need the following:

- 1 large piece of poster board with a 1 cm grid
- Multi-color Markers
- 1 cm by 20 cm strips
- Glue stick
- Grid paper
- Approximately 40 unit algebra tiles

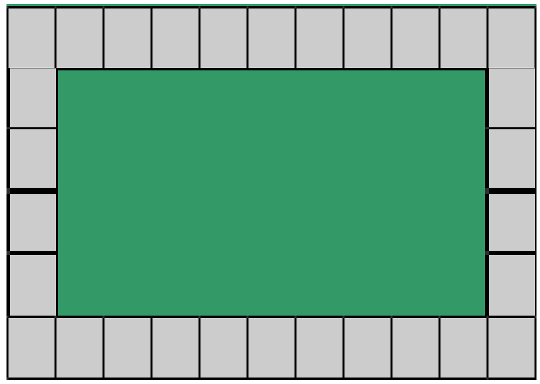
Steps

Introductory Activity**:**

1. Begin the day's lesson with a story, such as:
Last night, I saw the most wonderful garden. It had beautiful tiles all around it. So this morning, I asked my wife if we could install a garden in the backyard of my apartment. At first, she thought I was crazy, but I told her I'd make her a deal. I told her that if she landscapes my dream garden, I would install the tiles around the edges of the garden. So, she made a deal with me. She told me that she'd install a garden with an area of 36 square feet.
2. Ask the class, "If my garden has an area of 36 square feet, what are the possible dimensions of the garden?" Get from students all possible dimensions of the garden, using only whole numbers: 1 ft by 36 ft, 2 ft by 18 ft, 3 ft by 12 ft, 4 ft by 9 ft, and 6 ft by 6 ft.

3. Explain to students that you are on a budget, so you need their help in determining the least number of tiles that could be used around the outside edge of the garden. By using the Promethean White Board, display a 4 ft by 9 ft garden. Tell students that each algebra tile represents a 1 ft by 1 ft tile. Ask students to predict the number of tiles that would be needed to put a border of tiles around the entire garden.

4. On the board, record student guesses for the number of tiles needed. You may want to have the class reach a consensus regarding the number of tiles that will be necessary, or you may want them to discover this in their groups as part of the learning activities below. (For a 4 ft by 9 ft garden, the class should conclude that the border will consist of 30 tiles: the perimeter of the garden is 26 ft, and one tile is needed for each foot of perimeter; in addition, 4 tiles are needed at the corners, as shown below.)



***Learning Activities*:**

1. Explain to the class that they will be working in groups of four to investigate the number of tiles needed for gardens of various sizes. For the group exploration, provide the following directions:

- Sitting together, build gardens and make borders around the gardens.
- Record the number of tiles needed for each garden.
- Look for a pattern.
- Finally, come up with an algebraic expression that relates the length and width to the number of tiles needed.

Have students devise their own way of working together or you may want to assign the following roles to members of the group: writer, responsible for filling in the group's chart; cutter, responsible for the scissors; sticker, responsible for the glue; and speaker, who will present the group's findings to the class.

2. Assign one of the various garden sizes (from the introductory activity) to each group. The students in each group are responsible for constructing a model of the garden they are assigned. In addition, the group should consider all of the various garden sizes and look for a pattern that relates the length and width to the number of tiles needed.

3. Allow students time to construct a model of the garden they have been assigned. Students should cut the 1 cm by 20 cm strips to the length needed to form a border around their garden. Students may also use the 1 cm by 20 cm strips to investigate gardens of sizes other than the one they were assigned, or they can investigate using the grid paper. As students are working, circulate and use effective questions to help the groups identify the relationship between the length and width and the number of tiles.

4. After about 20 minutes, have each group present its findings. (Depending on the number of students in your class, this may mean that two speakers are presenting the same material, or it may mean that some sizes will not have been assigned.)

Students will invariably arrive at several different expressions for finding the number of tiles, including:

- $2l + 2w + 4$
- $2(l + w) + 4$
- $2(l + w + 2)$
- $2(l + 2) + 2(w + 2) - 4$
- $(l + 2)(w + 2) - lw$

5. After all student groups have presented their findings, describe one of the expressions that they have not discovered, and ask them to consider whether or not this alternative method is equivalent to their expression. For instance, you might say, "I was thinking that I would add the length and the width, double that result, and then add 4."

6. Select a student to translate your method into an algebraic expression. Be sure to discuss the order of operations.

7. Select another student to demonstrate how the equation found by their group is equivalent to the alternate expression that you suggested. (You may wish to repeat this step if several groups found different expressions. This discussion may allow for an explanation of the distributive property, the order of operations, the associative and commutative properties, and other topics.)

8. Ask again the question that was posed at the beginning of the lesson: "Which garden would require the fewest tiles?" Students should conclude that the 6 ft by 6 ft garden will only require 28 tiles, and that this is the fewest needed for any 36 sq ft garden.

Another extension of the problem would be ask how many tiles would be required if the width of the border is two (or more) tiles.

Then ask: "If I wanted a garden in which to grow different crops such as corn, tomatoes, green peppers, which would be the best one for several types of crops?" Students may suggest that the 1 ft by 36 ft garden is best, because it is the longest. Other students, however, will likely point out that such a garden would not be wide enough. Students may argue for the 2 ft by 18 ft and 3 ft by 12 ft gardens as the best candidates. While 4 ft by 9 ft and 6 ft by 6 ft would be wide enough for growing tomatoes, they would not be long enough for rows of corn.

Explain to students that because the sizes are not ideal for the dream garden, you would like them to consider other patterns for gardens. On the board or whiteboard, show them Draft 1, which is a 1 ft by 2 ft ; Draft 2, which is a 2 ft by 3 ft garden; Draft 3, which is a 3 ft by 4 ft garden; and Draft 4, which is a 4 ft by 5 ft garden. Ask them to use this pattern to predict what Draft 5 would look like, and then use their drawing to determine the number of tiles needed for the border of the Draft 5 garden. Similarly, have students determine the number of tiles needed for Draft 11, as well as for Draft *n*.

Allow students to present their findings to the class. In particular, encourage students to share their expression for the number of tiles. For Draft n, the length of the garden is $n + 1$, and the width is n. Consequently, numerous expressions could represent the number of tiles needed for the border of Draft n:

- $2(n + 1) + 2n + 4$
- $4n + 6$
- $2(2n + 3)$
- $2(n + 1 + n + 2)$
- $(n + 3)(n + 2) - n(n - 1)$

Have students use the expressions to confirm the number of border tiles for Draft 6 and Draft 11.

* In a students own words - Define and explain the components of the slope-intercept form of a linear equation. –steepness, slant of the line, etc…
* Use the slope-intercept form of a linear equation.
* Draw a diagram of the explanation.
* Discuss Slope (steepness).
* Explain how the equation of the line helps develop patterns to find future solutions to similar problems.

***Culminating Activity/Assessment*:**

Students might be asked to express their ideas regarding what they learned about algebra and the power of algebra. Allow several students to share their thoughts.

Related Standardized Test Questions

The questions below dealing with translating words into symbols have been selected from various state and national assessments. Although the lesson above may not fully equip students to answer all such test questions successfully, students who participate in active lessons like this one will over time develop the conceptual understanding needed to succeed on these and other standardized assessment questions.

□ Taken from the Maine Educational Assessment, Mathematics, Grade 11 (2002):

Carl needed to rent a car for one week. He collected the following information.

- Hillbrook Car Rentals charges \$230/week plus 20¢ per mile.
- Bridge Car Rentals charges \$210/week plus 25¢ per mile.
    a. Write an equation for the cost, c, to rent from Hillbrook Car Rentals if Carl drives the car x miles.

b. Write an equation for the cost, *c*, to rent from Bridge Car Rentals if Carl drives the car *x* miles.

c. For what number of miles will it cost the same to rent a car from the two companies? On what basis should Carl decide which company to rent from.

d. Show or explain how you found your answer.

Solution: For Hillbrook Rentals, $c = 230 + 0.2x$. For Bridge Car Rentals, $c = 210 + 0.25x$. The cost will be the same when the value of c is equal for both companies, which occurs when $230 + 0.2x = 210 + 0.25x$, or when $x = 400$ miles.

- Taken from the Maryland High School Algebra Exam (2002):

Lydia has $200 in her bank account at the beginning of the year. Each month, she deposits $40 into her account. She does not withdraw any money from her account, and the account pays no interest. Which of these equations could Lydia use to find the total amount (T) in her bank account at the end of *m* months?

A. $T = 40m$
B. $T = 240m$
C. $T = 200m + 40$
D. $T = 40m + 200$ (correct answer)

- Taken from the New York Regents High School Examination (January 2003):

The equation P = 2L + 2W is equivalent to

A. $L = \frac{P-2W}{2}$ (correct answer)
B. $L = \frac{P+2W}{2}$
C. $2L = \frac{P}{2W}$
D. $L = P - W$

- Taken from the Mississippi Algebra I Test, Version 2 (2003):

Jan took a taxi to visit the Petrified Forest northwest of Jackson, Mississippi. The taxi fare for the trip was $14.80, based on a fixed charge of $2.00 plus a charge of $0.80 for each mile. Which of these could be used to determine *m*, the number of miles that Jan rode in the taxi?

A. $14.80 = 2 + 0.80m$ (correct answer)
B. $14.80 = 2m + 0.80$

C. $14.80m = 2 + 0.80m$
D. $14.80m = 2m + 0.80$

 Taken from the Colorado State Assessment, Grade 5 (2002):

Which problem can be solved using the number sentence shown below?

$$6 \times 2 = \_\_$$

A. There are 6 clowns in a Volkswagon. Two more clowns are getting in. How many clowns are in the Volkswagon?
B. There are 6 children eating lunch. Each child ate 2 slices of cheese. How many slices of cheese were eaten? (correct answer)
C. There are 6 students playing basketball. Two left to get a drink of water. How many students are left playing basketball?
D. There are 6 students walking to the library. They are walking in groups of 2. How many groups of 2 are there?

Lesson Plan 2: Boxes and Blocks - Solving Linear Equations Using Manipulatives

Supplies**:**

Teachers will need the following:

- 40 Algeblocks (20 red, 20 blue)
- 10 cigar boxes or smaller
- 10 equations for use at stations (the equation should appear on one side of a strip of paper, and the solution on the other side)

Students will need the following:

- 20 blocks (red on one side, blue on the other)
- 10 cigar boxes
- Individual dry-erase boards or large sheets of paper

Steps

Introductory Activity**:**

1. As a warm-up, present the following equations for students to solve:

- $x + 8 = 14$
- $z - 4 = -2$
- $5 - t = -2$
- $y + 4 = -5$

2. Give students two minutes to complete the warm-up problems individually.

3. Have students compare and discuss their solutions with a partner.

4. For each problem, consider student answers. For any problem with which students had difficulty, ask several students with different answers to present their solutions on the board or overhead, and help them clarify their understanding.

***Learning Activities*:**

1. Distribute a Algeblocks, a set of boxes, and a large sheet of paper or dry-erase board to each group of students.

2. Explain that students will be using a boxes and blocks activity to solve the equation $2x + 6 = 12$.

3. Present the following directions to students:

- If the variable is positive, place the box(es) facing up.
- If the variable is negative, place the box(es) facing down.
- The coefficient of the variable indicates the number of boxes to use.

Then, ask students to show you the representation of $2x$ using the boxes. They should all place two boxes facing up on top of their paper or dry-erase board. Explain the following:

- The blocks represent the numbers.
- If a number is positive, the block should be blue side up.
- If a number is negative, the block should be red side up.

Have students use six blue blocks to represent +6. They should place these blocks next to their two boxes. Then, have them draw an equal sign to the right of the two boxes and six blue blocks. Explain that they can represent +12 by placing 12 blue blocks on the other side of the equal sign.

4. Ask students what can be done to both sides of the equation to get rid of the six blue blocks (+6) on one side of the equation. Elicit from students that -6 should be added to each side (i.e., add six red blocks to both sides); alternatively, +6 could be subtracted from each side (i.e., take away six blue blocks from each side).

5. On the overhead, add six red blocks to the side with six blue blocks. Also add six red blocks to the side with 12 blue blocks, and have students repeat these actions in their groups. Ask, "When you pair a red block with a blue block, what happens?" Call on a student to explain that each such pair is equal to 0.

6. Have students remove the pairs of red and blue blocks, leaving just two boxes facing up and six blue blocks. Ask, "What equation do we have now?" Elicit from students that the boxes represent $2x$, the remaining blue blocks represent +6, and the equation now left is $2x = 6$. Write this new equation on the overhead below the original equation.

7. Ask, "If two boxes equal six blocks, what does that tell us about one box?" They should notice that there are three blocks for each box.

8. Demonstrate that the final equation is now $x = 3$, and write this equation on the overhead below the equation $2x = 6$.

9. Give students the following problems to solve in their groups using boxes and blocks:

- $5m + 1 = -9$
- $2x + 3 = 4$

10. Circulate as students are solving these problems. Allow a few minutes for students to complete both problems.

11. Review the solutions to the problems with the class. For the second problem, be sure to discuss the final step, when students arrive at the equation $2x = 1$. Ask, "Were you actually able to use the boxes and blocks to solve the problem? When you had $2x = 1$, what operation did we have to do?" Elicit from students that both sides had to be divided by 2 (or that the block needed to be split in half), to yield the answer $x = ½$.

12. Explain to students that you want them to try a problem with a negative coefficient. Give students the problem $-2x + 3 = -5$ to solve.

13. Ask, "What was the first step in solving this problem?" The students should notice that the first step is to subtract 3 from (or add -3 to) both sides of the equation, yielding -$2x = -8$.

14. Ask, "What is the next step to balance the equation and get x by itself?" Students may note that both sides need to beyielding $x = 4$. They may also state or demonstrate that they can turn over both the boxes and the blocks on both sides of the equation, which would represent multiplication by -1.

15. Ask, "How can we check this to make sure it is the correct answer?" Obtain from students that the value $x = 4$ can be substituted into the original equation to show that it works: $-2(4) + 3 = -5$.

Explain to students that now that they have solved the same equations using boxes and blocks and symbolic manipulation (or algebra), it's time to try solving similar equations with symbolic manipulation (algebra) only. At 10 stations throughout the room, post various equations for the students to solve. Do not let them know that the solutions are given on the back of each piece of paper. Have students circulate in pairs through the stations, solving each equation and checking their answers. Give students 1-2 minutes at each station, as necessary. Below are some equations you might use (make sure some of the variables have negative and fractional coefficients):

- $3x + 2 = 14$
- $-3m - 1 = -10$
- $-7x + 5 = 12$
- $-w + 13 = 9$
- $\frac{1}{2}d + 7 = 10$

16. Show students that they can turn over the papers to find the correct solutions. Give them a couple of minutes to verify their results, and then call the whole class together to review and clarify the solutions to any problems with which students had difficulty.

***Culminating Activity/Assessment*:**

1. Once students have answered all questions, ask them to summarize the process of solving an equation. Solicit input from several students, and relate their descriptions to the boxes and blocks activity. Emphasize the need to add or subtract and then multiply or divide, and be sure to stress that the final step should always be to check the answer in the original equation.

2. Assign problems for homework.

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