Great Outcome

Carmen Kissoon-Ragoonanan
Beeber Middle School

Overview

The National Council of Teachers of Mathematics (NCTM) has developed five content standards and one of these standards is Data Analysis and Probability. NCTM advocates have placed tremendous emphasis on data and probability because of the data that we have to face in every day life. In teaching students to deal with data, the Standards document observes that "ideas from probability serve as a foundation to the collection, description, and interpretation of data" (Standards for School Mathematics).

In order for students to develop probabilistic thinking in the middle grades, NCTM advocates have recommended that students should be given many opportunities to make predictions and explore to find solutions. By exploring, they can figure out how close their guess or predictions were to the actual event occurring. With today's advancement in technology and the use of computers in classrooms, students should be given the opportunity to use computer simulations to generate large samples of data which can be analyzed in a short period of time. However, performing the actual experiments can help to develop students' thinking and help teachers to develop questions and discussion needed to explore probability concepts.

Students' performance on the state test has usually been low and probability is one of the concepts being assessed. Teachers will argue that students' performance is low because we are provided with a time line that we have to follow and enough time is not spent on building a concept in the classroom. Based on my experience, I believe that teachers are unable to teach this concept before the state test because they have to follow the pacing guide provided by the school district and this concept was meant to be taught after testing. Teachers are also given a mandated curriculum and this unit can be used to enhance the curriculum provided. Some of my colleagues will argue that the mandated curriculum should be enhanced with outside resources. This will help to motivate students to explore the same concept using a different method and exploring various problems apart from those taken from the text book and applying critical and reflective thinking skills to solve probability problems.

Critical and reflective thinking skills will be used to help students solve problems in this unit. Critical thinking can be defined as the use of cognitive skills or strategies that increase the probability of a desirable outcome that is reasoned and goal directed – the kind of thinking involved in solving problems, formulating inferences, calculating likelihoods, and making decisions when the thinker is using skills that are thoughtful and effective for the particular context and type of thinking task (Halpern 1996). Critical thinking can be defined as a wide range of thinking skills, which is a part of reflective thinking. According to Dewey, that reflective thinking is an active, persistent, and careful consideration of a belief or supposed form of knowledge, of the grounds that support that knowledge, and the further conclusions to which that knowledge leads. Learners are aware of and control their learning by actively participating in reflective thinking – assessing what they know, what they need to know, and how they bridge that gap – during learning situations (John Dewey 1993).

In order to have students use critical and reflective thinking skills, this unit will include a bank with problems of the week. These problems will include the use of a newspaper article as an introduction to the unit. For example, students might estimate the chances of Dan Jansen winning a medal in the winter Olympics. Other concepts covered in this unit include dependent and independent events occurring, and the use of tree diagrams or the drawing of a picture to solve probability problems.

This unit is written for students in the sixth and seventh grades. These students have forty-five minute math periods on a daily basis. I intended to incorporate this unit into the Prentice Hall mathematics program, which is currently used in my classroom. In order to prepare this unit I have researched the NCTM standards and their approach to solve probability problems in the classroom. Students will explore various strategies and use the one that they are comfortable with to find the solution and become critical and reflective thinkers. Emphasis will also be placed on defining vocabulary words related to each lesson and the use of the four square approach to define these (Frayer's Model).

The following schedules are suggested for incorporating this unit into the schools' curriculum. Teachers can adapt these schedules to fit into their teaching block or curriculum.

1) Monday- 25 minutes
2) Tuesday, Wednesday, Thursday, and Friday – 20 minutes each with probability used as independent work and students working in small groups.

Rationale

My school is now one of the empowerment schools with low performing students. I am currently teaching three groups of students who are performing at different levels. My lowest group can be described as functional third grade students. Probability is one of the concepts being tested on the state test and I am concerned about their performance in this area. The purpose of writing this curriculum unit is to explore probability problems taken from various sources and differentiate instructions to enhance students' learning and performance based on real life experiences. Some problems in this unit can be solved using a formula while other problems will help students to develop critical and reflective thinking skills. Students will also be allowed to choose a solution strategy, improve their test scores and help to close the math gap within the district. Many of the problems were taken from the PSSA Assessment Anchors and are geared towards preparing students for mastery on the state test. This unit consists of a probability problem bank with problems of the week. Teachers can start out with simple probability problems then have students begin solving more difficult problems. Teachers can assign these problems of the week for homework assignments or class-work.

Objectives

How are statistics used in probability?

Statistics is most often used to describe the behavior of a person or thing. For example, statistics are used to describe the performance of an athlete. A baseball player can be a .300 hitter while another team member can have a .250 batting average. Based on this statistic, a baseball coach can help to figure out which player needs additional help in improving their hitting.

Statistics can influence a person's decision making whenever there is uncertainty about the things involved. For example, statistics are used to determine some of the things in real life situations such as the rate to charge for insurance, the length of product warranties and to track the performance of good sport players.

Probability is a way of expressing knowledge or belief that an event will occur or has occurred. According to this definition, all decision-making has some probability in it, even real life situations (Fundamentals of Probability, Statistics, Experiments and Data page 1).

Variables and Measurement

One of the vocabulary words for students to become familiar with is variable. A variable can be defined as something that can change. If a coach wanted to determine the batting average of one of his baseball players he would have to take into consideration two variables: 1) the number of hits, and 2) the number of times at bat. To calculate the batting average of a baseball player, the coach must observe the desired behavior, the ability to hit the ball successfully, and to take measurements of the behavior. In this case, the ability to hit the ball is based on the measurements of two variables, the number of times the baseball player is batting and the number of times the baseball player is able to hit the ball successfully (Fundamentals of Probability, Statistics, Experiments, and Data page 2).

There are four different kinds of measurements. These include:

1) Nominal measurement – These numbers are placeholders, and they can only be replaced by names. Examples of nominal measurements can be the type of pies served in a food court. These can include chocolate pie, banana cream pie, and apple pie.
2) Ordinal measurement – These measurements have rank or order. They can be compared to find out if one is greater than the other. Examples of ordinal measurements include the finish position in a horse race. In a race, horses finish in a sequence, which is first place, second place, and third place.
3) Interval Measurement – This is based on a standard scale and can be compared to see how many units are greater. An example of interval measurement is temperature in degrees Celsius.
4) Ratio Measurement - This has interval measurements with ratios between the possible ratio values. Examples of ratio measurements are the gallons of water used in flushing a toilet or the number of slices of pizza eaten (Fundamentals of Probability, Statistics, Experiments, and Data page 3).

In this unit, students are required to use critical thinking skills and reflective thinking to solve problems and finds solutions to probability problems assigned. According to research studies these are the characteristics of environments and activities that prompt and support reflective thinking:

- provide enough wait time for students to reflect when responding to inquiries
- provide emotionally supportive environments in the classroom encouraging reevaluations of conclusion
- prompt review of the learning situation, what is known, what is not yet known and what has been learned
- provide authentic tasks involving ill-structured data to encourage reflective thinking during learning activities
- prompt students’ reflection by asking questions that seek reasons and evidence
- provide some explanations to guide students’ thought processes during exploration
- provide a less structured learning environment that prompts students to explore what they think is important
- provide social learning environments such as those inherent in peer groups and small group activities to allow students to see other people’s point of view (Reflective Thinking page 1).

Lessons from this unit will be assigned to students placed in small groups allowing them to work collaboratively. Ideas from this research will be applied to the classroom environment. When a problem is assigned a K-W-L chart will be completed by students. A K-W-L chart is a graphic organizer that helps to scaffold concepts that students need to understand. This allows the teacher to find out what students know about the problem, highlight key words in the problem to find out what they want to know, and after finding their solution they reflect on what they have learned. Each group will be required to share their solutions and this will generate discussion based on strategies and solutions and difficulties they have in finding solutions.

The objectives of this probability unit will include the following:

- to investigate chance and expected outcomes in real world situations in which the outcome is uncertain
- use cross tables and tree diagrams to represent data for possible events
- express the probability of a simple event as a fraction, decimal, and percent
- list the possible outcomes of two independent events and compare the outcomes
- find and interpret the experimental probability of an outcome of a simple event
- find solution, show work, and write an explanation to solutions for open-ended questions.

Strategies

The following strategies will be incorporated into students' inquiry approach.

The Four Square Approach

The four square approach is a graphic organizer that can be used to enhance a lesson. This model is frequently used to introduce vocabulary words. New concepts for this unit will include combinations, permutations, sample space, event, and outcome. Using Frayer's model of the four square approach is one of the suggestions for building vocabulary words. In the first square, students will write the definition of the word; in the second square, they will give a non example of the word; in the third square, they will give an example of the word; and in the final square, they will draw a picture of the word or write a formula.

K-W-L: What you already Know, What you Want to know, and What you Learned

The K-W-L chart is another graphic organizer that can be used to activate prior knowledge of what students know about probability. This can also be used to find out what students know about a particular problem and to stimulate their thinking about what they want to know about the problem. The final column can be filled out at the end of a lesson or after the solution of a problem to summarize what students have learned.

Draw a picture or Diagram

The majority of students in my third group are visual learners. The use of charts, tables, tree diagrams, and pictures will help students to find the solution to a problem. This will help them to explain their reason for using a picture or chart. They can also incorporate this into their explanation for constructed response questions included in this unit. They will be required to use the magic words in their explanation. Examples of magic words include the following: (to get, to see, to find, to show, to figure out, because, since)

Constructive Response

The PSSA, which is a standardized test, requires students to complete constructive response questions. Teachers are required to use TAG it a 3 to teach constructive response questions. Students are required to turn the question in a math problem into an opening statement, and use numbers /words to explain their solutions.

T-Charts

T-Charts can be used to solve open-ended questions. This is a two-column chart. The first column is used to show students' work and the second column is used to write a step-by-step explanation to their solution. T-Charts are perfect in helping students to organize their solution and explanation. The goal of constructed response questions in this unit is for students to apply their knowledge from probability concepts taught and to develop critical thinking skills.

The Hand

The hand is another graphic organizer that students can use to ensure that all parts of the question in a constructed response problem are answered. This graphic organizer is sometimes incorporated into my lesson. Each finger represents a step in problem solving. Starting from the little finger (pinkie) these are the five steps to problem solve:

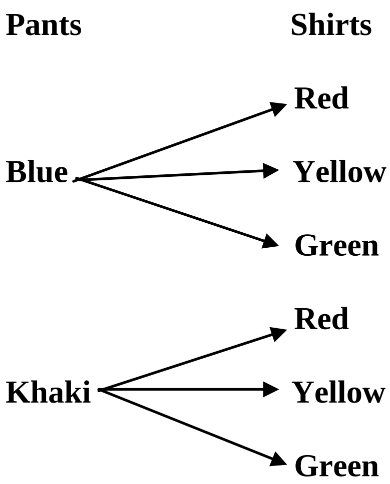
1) Underline the problem
2) Circle the facts
3) Choose a strategy
4) Solve the problem
5) Does your answer make sense?

Tree Diagrams and Counting Principle

Tree diagrams can be used to show a list of possible outcomes in an organized fashion. This uses branches to show individual choice or stage. This uses the counting principle because a branch is needed to show each stage. The total number of outcomes is the product of the number of outcomes at each stage (Prentice Hall Mathematics).

Here is a problem that a tree diagram can be used to show a list of all possible outcomes.

You can use blue or khaki pants and a red, yellow, or green shirt. Find the sample space for this problem using a tree diagram. How many outfits are possible?



There are six possible outcomes.

You can use the counting principle to find the number of ways to make one choice followed by a second choice. For example, suppose there are x ways of making one choice and y ways of making a second choice. Then there are x * y ways to make the first choice followed by the second choice (Prentice Hall Mathematics).

Here is an example using the counting principle:

You can use one item from each category in the menu below.
How many different desserts can you order?

Ice Cream Menu

Flavors	**Toppings**	**Cones**
Vanilla	Nuts	Waffle
Chocolate	Sprinkles	Sugar
Strawberries	Cherries	
Banana		
Peach		

Solution:

Flavors		Toppings		Cones		Desserts
5	$\times$	3	$\times$	2	=	30

You can order 30 different desserts.

In the menu above, cherry ripple ice cream as a flavor and fudge as a topping are added. Find the new number of different desserts.

Solution:

Flavors		Toppings		Cones		Desserts
6	$\times$	4	$\times$	2	=	48

Probability

In mathematics, probability can be expressed as a number between 0 and 1. This gives an estimate of the occurrence of the frequency of an event. Probabilities can be written as fractions, decimals, and percents. The formula below can be used to find the probability of an event occurring.

Probability of an event $$P\text{ (event)} = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$$

Here is an example of the probability of an event:

You roll a number cube once. Find the P (an even number)
There are three outcomes out of a total of six likely outcomes. The even numbers are 2, 4, and 6. The possible numbers on the number cube are 1, 2, 3, 4, 5, and 6. P (even) =3/6
3 are the number of outcomes with even numbers and 6 is the total number of outcomes. 3/6 when simplified is = ½ (Prentice Hall Mathematics).

Whenever the probability of an event occurring is 0, the result is impossible. When the probability of an event occurring is 1, the event is certain to happen.

Experimental Probability

The results of an actual experiment occurring are experimental probability. This is based on data that have been collected. Theoretical Probability is based on the assumption that certain outcomes are equally likely. The formula below can be used to find experimental probability.

$$P\text{ (event)} = \frac{\text{number of times an event occurs}}{\text{Total number of trials}}$$

Here is an example of an experimental probability problem:

In 20 tennis matches against Jennie, Ai-Ling wins 9 times. What is the experimental probability that Ai-Ling wins a match?

P (Ai-Ling wins) = 9/20 $$\frac{\text{Number of matches Ai-Ling wins}}{\text{Total number of matches}}$$

The experimental probability that Ai-Ling wins a match is 9/20.
Another question could be added to this problem, which is what is the experimental probability of Jennie winning the match? The solution is 11/20.

Analyzing Experimental Probability

Fair Games
You and your friend want to play a game, but your only number cube is chipped. To make a fair game, you roll the number cube 60 times. The results are displayed in the table below.

Outcome	1	2	3	4	5	6
Number of times rolled	16	17	12	8	4	3

Which of the following games seems fair? Explain.

a) You win if the number rolled is 6. Your friend wins if the number rolled is 2.

P (6) = 3/60 There are three rolls of 6 in 60 trials.

P (2) = 17/60 There are 17 rolls of 2 in 60 trials.

You would expect P (2) and P (6) to be about the same with a fair number cube. Since this number cube strongly favors 2, the game seems to be unfair.

b) If the number rolled is even, you win. If it is odd, your friend wins.

17 + 8 + 3 = 28 Add to find the number of even rolls
16 + 12 +4 = 32 Add to find the number of odd rolls.
P (even) = 28/60 There 28 even rolls in 60 trials
P (odd) = 32/60 There are 32 odd rolls in 60 trials.

You would expect p (even) and P (odd) to be about the same with a fair number cube. The probabilities are about the same, so the game seems to be fair (Prentice Hall Mathematics).

Mathematical Understanding of Probabilities

Probability can help you make predictions about the outcome of an experiment. However, there is no guarantee of the actual outcome of an event. The probability of an event may not be a good predictor of what will happen in a small number of cases. The laws of probability will predict the occurrence of an event when you survey a very large number of events (Prentice Hall Mathematics).

Independent events

When the occurrence of event 1 does not affect the occurrence of event 2, the two are independent events. A formula can be used to calculate the probability of an independent event.

If A and B are independent events, then P (A and then B) = P (A) $\times$ P (B)

An example of an independent event is when a person roll a six-sided number cube twice, the probability of rolling a 2 followed by a 2 is 1/6 $\times$1/6, or 1/36.

A sample space can be defined as the set of all possible outcomes of an event. Here is an example of finding a sample space.

Use the menu that is given to find the sample space for the selection of a main dish and a side dish.

Lunch Menu

Main Dish	- Grilled chicken
	- Baked Chicken
Side Dish	- Salad
	- Vegetable
	- Rice

How many possible outcomes are there? List all the possible outcomes.

Grilled chicken	salad	Baked chicken	salad
Grilled chicken	vegetable	Baked chicken	vegetable
Grilled chicken	rice	Baked chicken	rice

The number of possible outcomes is 6.

Here is another example of finding a sample space.
A bicycle comes in three models and three colors. Construct a sample space to find how many bicycle choices you have. Find the number of possible outcomes.

Model 1, color 1
Model 2, color 2
Model 3, color 3
Model 2, color 1
Model 2, color 2
Model 2, color 3

Model 3, color 1
Model 3, color 2
Model 3, color 3   (Prentice Hall Mathhematics page 476)

Classroom Activities/ Lesson Plans

This unit starts out with simple probability problems for students to solve. As they work through these problems, they will apply what they learn from each problem to solve more difficult problems.

Some constructed response items are included in this unit. Students can be prompted to write a thorough explanation to their solution. It is recommended that problem1 should be used as an introduction to the probability unit. This problem will help students to define probability and to make predictions about chances in real world events.

The four square approach can be used to define the following terms: experiment, sample space, tree diagram, outcome, event, probability, fair, and unfair. At the beginning of each lesson at least one word can be defined and other terms that apply to the lesson can be discussed.

<u>Lesson 1</u>
Duration of time: 20 – 25 minutes
Directions: It is recommended that the Dan Jansen story be used as an introduction to this unit.

Problem 1
Will Dan Jansen Win?

The following article appeared in a newspaper during the 1994 winter Olympics.

Dan Jansen is considered by many to be the world's best 500-meter speed skater, now or ever. But he finished eighth in that event Monday.

He is not the world’s best 1,000-meter speed skater, but that distance is his last chance at an Olympic medal after seven races in four Olympics.

A brief survey of Olympic history indicates that his chances in Friday’s 1,000 meter are not good:

He finished 16th at 1,000 meters after a fourth place 500 in 1984.
He fell at both distances in 1998.
He finished 26th at 1,000 meters after a fourth place 500 in 1992.
But his coach, Peter Mueller, says Jansen’s chances Friday are good.

Even though the newspaper article predicted otherwise, Dan Jansen won the Olympic gold medal in the 1,000-meter race at the 1994 winter Olympics. Even more impressive, his time at 1: 12: 43 was a new world record for the event.

1a. Give some reasons why making predictions like the one in the newspaper article is difficult?
b. This article mentioned only two levels of chance, “not good” and “good.” Design a scale that has more levels.

Solution 1
1a. Making predictions like the one in the newspaper article are difficult because an athlete’s performance can be affected by several conditions. Some of these can include the following:

- The weather conditions can affect an athlete’s performance
- A skater suffers an injury from practice.
- A skater can fall and this can interfere with another athlete’s performance.
- It is difficult to judge a skater’s physical condition.

1b. Students can use words such as very good, good, average, below average, and poor to describe the scales that are developed. Percents can be used to describe the various levels of their scales. Percents will include 0%, 25%, 50%, 75%, and 100%. These percents can also be converted into decimals. Scales can be displayed using number lines (Math in Context).

Extension

Usain Bolt is a Jamaican sprinter. He holds world and Olympic records in both the 100 meters and 200 meters, with times of 9.69. Discuss the conditions of this athlete and predict his performance in the next Olympic in the 100 meters and 200 meters. Students can also complete research using the Internet and compare his performance to the times of other sprinters in the Olympics.

Lesson 2
Duration of time (20 – 25 minutes)
Directions: For each problem define and /or find the probability of a simple event (express as a fraction in lowest terms).

Problem 2-1
There are 12 boys and 8 girls in Ms. Chowdbury’s class. Ms. Chowdbury writes each student's name on a piece of paper and drops it in a paper bag. If she closes her eyes and picks a name without looking, what is the probability that she will draw a boys’ name?

Problem 2-2
Reggie will roll a number cube with faces 1 to 6. What is the probability that he will roll a number greater than 3?

Solution 2-2

Computer Simulation

The link below can be used as an extension. Teachers can extend problems on the number cube and have students figure out the probability of rolling two number cubes and getting two numbers that add up to 6, 7, or 8.

Rolling Dice

Problem 2-3
There are 13 white tiles, 12 gray tiles, and 15 black tiles in a box. If Quincy reaches in and picks a tile at random, what is the probability that he will choose a gray tile?

Solution 2-3

Problem 2-4
The letters of PENNSYLVANIA were written on index cards and placed inside a brown paper bag. Pilar will pick one of the letters from this bag without looking. What is the probability that she will pick the letter N from the bag?

Solution 2-4

Problem 2-5
Lloyd rolls a number cube, with faces numbered 1 to 6. What is the probability that he will roll a number greater than zero?

Solution 2-5

Problem 2-6
There are 50 unlabeled movie DVDs in a bin at a video store. Seven are comedies, 10 are musicals, 13 are dramas, and 20 are action films. What is the probability of reaching into the bin and randomly selecting a comedy?

Solution 2-6

Problem 2-7
This list shows the names of 25 students entered in a contest to win a free dinner at a local restaurant. Each student's name is written on a piece of paper and placed in a box. If a name is drawn at random, what is the probability that the name picked start with an M?

Jim	Felicia	Daryl	Matt
Justine	Delia	Sandra	Donald
Patrick	Beth	Sam	Marty
Shinah	May	Ned	Alan
Holly	Ping	Marco	Edwin
Tucker	Vicky	Gary	Oatis
Cindy			

Solution 2-7

Lesson 3
Duration: 20 – 25 minutes
Directions: Determine/show all possible combinations involving no more than 20 arrangements (e.g. tree diagram, grid, table).

Problem 3-1
How many different four digit numbers can be made using the digits 6, 6, 7, and 8, if the digits 7 and 8 are used only once?

Solution 3-1

Problem 3-2
If you order a sandwich at the sandwich shop, you can choose from two different kinds of bread: white or pumpernickel. You can then choose from four different kinds of meat: turkey, ham, roast beef or chicken.

If a sandwich consists of one kind of bread and one kind of meat, how many different sandwiches could you order?

A tree diagram can be used to show the solution. An organized list can also be used.

Solution 3-2

Problem 3-3
Hal has three different books. How many different ways can he arrange the books side by side on a shelf?

Solution 3-3

Problem 3-4
Katrina has 2 hats (one red and one purple) and two scarves (one black and one gray). How many different combinations of a hat and a scarf could she wear? Use a tree diagram or a table to find all the possible combinations of hat and a scarf.

Solution 3-4

Problem 3-5
How many different 3-digit lock combinations can be made using the digits 3, 3, and 7 if the digit 7 is used only once?

Solution 3-5

Problem 3-6
An out fit consists of a shirt and a pair of pants. How many different outfits can be made using 3 different shirts and 3 different pair of pants?

Solution 3-6

Problem 3-7
Antonio has 4 textbooks: a history book, a math book, a Spanish book, and a health book. How many ways can he arrange these books side by side on a shelf if the history book is always the first book on the shelf?

Solution 3-7

Problem 3-8
There are four students running for school president and 3 students running for school vice -president. How many different combinations of a president and a vice president could there be?

Solution 3-8

Problem 3-9
Reese orders soup and bread at a restaurant. He can choose from 3 different kinds of soup: tomato, potato, or vegetable. He can then choose from 3 different kinds of bread. How many different combinations could he order?

Solution 3-9

Problem 3-10
There are 4 routes from Danna's house to her school. There are 5 different routes from her school to the library. If Danna goes from her house to the school and then to the library, how many different routes could she take?

Solution 3-10

Problem 3-11 (Constructed Response)
What's My Number?

I moved to a new town and asked the phone company what my new number would be. I was told these conditions apply to new phone numbers in my area:

The first three digits are 946.
The fourth digit is odd and less than 5.
The fifth digit is even and less than 5.
The sixth and seventh digits are even.

How many possible phone numbers are there using these conditions? Explain your solution.

Solution 3-11

Lesson 4
Duration: 20 – 25 minutes
Directions: Divide class into small groups. Have students use what they have already learned about probability to solve any of the problems from lesson 4.

Problem 4-1
Ms. Logan, the music teacher, can take one-sixth-grade student and one-seventh-grade student to a classical music concert on Friday. If 3 sixth grade students and 5 seventh grade students would like to go, how many possible combinations of a sixth grade student and a seventh grade student could she take to the concert?

Solution 4-1
There are 15 possible combinations of a sixth grade student and a seventh grade student that can be taken to the concert (PSSA Coach Assessment Anchors).

Problem 4-2
A bag contains only red, green, and blue marbles. P (Green) = 1/3, and P (red) =1/2. There are 6 green marbles. How many blue marbles are there in the bag?

Solution 4-2

Problem 4-3
Alicia gets her hair braided at a salon. She has a choice of 4 styles of braid, 5 ribbon colors, and 3 barrettes. Alicia may choose one style of braid, one ribbon color, and one barrette. How many different hairdos can she get?

Solution 4-3

Problem 4-4
A bag contains 3 red markers, 4 green markers, 2 purple markers, 2 black markers, and 1 blue marker. You pick a marker at random. Find the probability of each event. Write each answer as a fraction, a decimal, and a percent.

P (green)

P (not purple or blue)

P (pink)

P (blue)

Solution 4-4

Problem 4-5
You have three shirts and two pairs of jeans. Construct a sample space of the possible outfits you can wear.

Solution 4-5

Problem 4-6
A package of 25 party favors contains 8 glitter balls. A package of 20 party favors contains 6 glitter balls. You randomly select a party favor from each package. In which package are you more likely to get a glitter ball as a party favor? Explain your solution.

Solution 4-6
A package of 25 party favors because there are more glitter balls in the bag (Prentice Hall Mathematics).

Problem 4-7
A baseball team has the starting and relief pitchers shown in the table. The manager selects a pitcher at random. Find the probability that the pitcher is left -handed.

Pitchers on baseball team

Pitchers	Numbers
Left-handed starters	1
Right-handed starters	4
Left-handed relievers	2
Right-handed relievers	1

Solution 4-7

Problem 4-8
A clown has 21 balloons to sell. Seven of the balloons in the bunch are yellow. You take the string of a balloon at random. What is the P (yellow)? What is the P (not yellow)?

Solution 4-8

Problem 4-9
Lainie wrote the letters of the word MATHEMATICS on index cards as shown.

M T E A I S
A H M T C

She then shuffled the cards and picked one at random. What is the probability that she picked the letter M?

Solution 4-9

Lesson 5
Duration: 20 – 25 minutes
Directions: Divide students into small groups and have them choose one of the constructed response questions from this section and solve. Share strategies and solutions.

Problem 5-1
The Monty Hall Problem - *Let's Make a Deal!*

Imagine that the set of Monty Hall's game show *Let's Make a Deal* has three closed doors. Behind one of these doors is a car; behind the other two are goats. The contestant does not know where the car is, but Monty Hall does.

The contestant picks a door and Monty opens one of the remaining doors, one he knows doesn't hide the car, if the contestant has already chosen the correct door, Monty is equally likely to open either of the two remaining doors.

After Monty has shown a goat behind the door that he opens, the contestant is always given the option to switch door. What is the probability of winning the car if the contestant stays with her first choice? What if she decides to switch?

One way to think of this problem is to look at the sample space. Monty changes this by opening one of the doors that has a goat behind it. In doing this, Monty effectively eliminates one of the two losing doors from the sample space.
The assumption can be made that there is a winning door and that the two remaining doors, A and B, both have goats behind them. There are three options:

1. The contestant first chooses the door with the car behind it. She is then shown either door A or door B, which reveals a goat. If she changes her choice of doors, she loses. If she stays with her original choice, she wins.

2. The contestant first chooses door A. She is then shown door B, which has a goat behind it. If she switches to the remaining door, she wins the car. Otherwise, she loses.

3. The contestant first chooses door B. She is then shown door A, which has a goat behind it. If she switches to the remaining door, she wins the car. Otherwise, she loses.

Solution 5-1

Problem 5-2
Candy Arrangements

I opened a bag of candy with 12 licorice bits in it. There were 9 red pieces and 3 black pieces. I decided to arrange them in a circle with no two black pieces together and I wondered how many ways this could be done. I found three arrangements, and then I realized that they were all the same arrangement. Each one could be rotated or flipped to match the others. How many unique ways are there to arrange the candy around the circle? What are they?

For struggling students, teachers might want to give them counters (nine of one color and three of another color) to represent the candies. Draw a circle with 12 spaces in which to place the counters. Fill in the counters. Draw this arrangement or record with letters. Rearrange them and record. Continue to do this until you think that you have found all the arrangements. Remember that an arrangement that is rotated around the circle is still the same.

Think of the 12 spaces in the circle being arranged in a line and focus on the three black candies. If they were in a circle, any rotation of the same arrangement would still be the same arrangement. This means that by keeping track of the same number of spaces between the blacks, we can find all the arrangements. For example, two arrangements are shown below.

1	2	3	4	5	6	7	8	9	10	11	12	SPACING
B		B		B								1, 1, 7
B		B		B								2, 1, 6

Solution 5-2

Problem 5-3
There are 2 blue marbles, 3 red marbles, and 5 green marbles placed in a bag.

What is the probability of reaching into the bag without looking and drawing a blue marble? (Be sure to give your answer as a fraction in simplest form.)
Use words and / or numbers to explain how you found your answer for Part A.

Solution 5-3
1/5 - There are 10 marbles, so there are 10 possible outcomes. There are 2 blue (B) marbles, so there are 2 favorable outcomes.

Probability is the number of favorable outcomes over the number of total possible outcomes, so the probability is 2/10, 2/10 simplifies to 1/5, so the final answer is 1/5 (PSSA Coach Assessment Anchors).

Problem 5-4
Luann wants to wear a blouse and a skirt for class picture day. She can choose from 5 different blouses: a red blouse, a purple blouse, a gray blouse, an orange blouse, or a yellow blouse. She can choose from 2 different skirts: a black skirt or a white skirt.

How many different combinations of a blouse and a skirt could she wear for class picture day? Draw a tree diagram or a table to show how you determined all of the possible combinations.

Solution 5-4

Sometimes students enjoy choosing their own problem to find a solution. It is recommended that students be given this opportunity to choose problems from the bank then solve and share solutions. However, each student should complete all the constructed response questions at some point. Questions that are not completed in class should be assigned as homework problems.

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This unit was once a part of the schools' curriculum. The newspaper article from this book can be used as an introduction to the unit. This is based on the prediction of Dan Jansen. .

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The Math Forum @ Drexel http://mathforum.org/dr.math/faq.monty.hall.html
The Monty Hall Problem was taken from this source.

Prentice Hall Mathematics. Copyright @ Pearson Education, Inc.
More practice problems based on dependent and independent events was taken from this source.

Pennsylvania PSSA Assessment Anchors. Mathematics, Grade 6. @ 2007 Triumph Learning, LLC
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Study Outline_ Chapter Twelve: Probability
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Teacher Resources

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Student Resources

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Appendix

Standards

Standard 2.7 Probabilities and Prediction

2.7.6.A Collect data and estimate the likelihood of outcomes of events.

2.7.6.B Organize data collected in a simulation and select an appropriate format to display the data.

2.7.6.C Express the probability of a simple event as a fraction, decimal, and percent.

2.7.6.D List the two possible outcomes of two independent events and compare the outcomes.

2.7.6.E Find and interpret the experimental probability of an outcome of a simple event.

Assessment Anchors

M.6.E.3 Understand and apply basic concepts of probability.

M.6 E.3.1 Determine / show all possible combinations. Outcomes, and / or calculate the probability of a simple event.

M.6.E.3.1.2 Determine / show all possible combinations involving no more than 20 arrangements (e.g. tree diagram, table, grid).