The Games We Play – An Interactive Analysis of Probability

Brian Cohen
High School of the Future

Overview: Probability plays a crucial role in the decisions individuals make on a daily basis: we determine which route is potentially fastest to get to work; we choose which restaurant to eat at in order to avoid lines. Unfortunately, our students do not always understand how to interpret those situations correctly in order to make the proper choice. This abstract concept of probability in choice is difficult to grasp unless the students have a basis in counting methodology and concrete examples, like games. Additionally, when discussing probabilistic situations, students easily get confused by the use of percent, fraction, and decimal descriptions. Games provide rich subject matter to draw from when discussing this difficult subject.

This unit will use probability to analyze student-created games (board, card, dice, and other) and connect that concept to situations they have to deal with on a daily basis (as well as in the future). Students will focus on counting the various possible outcomes in their games and will calculate the probabilities associated with each. They will also be provided with well-known games in order to gain an understanding of how to analyze real-world situations. International probabilistic games will be showcased in order to promote the idea that similar math concepts exist all over the world and can be analyzed the same way. Students will then be provided with certain scenarios (ex. disease testing) and asked to use their knowledge choose what will provide the best outcome.

They will gain more proficiency in using percents, fractions, and decimals by analyzing and discussing the different games they will make and encounter. They will convert between the different representations of numbers and become comfortable using the vocabulary. This skill will allow them to understand and explain why different games are more or less likely to cause a favorable outcome for the player. They will also help the student explain the major differences between probability in games and in other situations they might experience in their own lives.

While these topics are useful and practical for many different grade levels this unit will focus on the high school level of education. The experience will be rich with technology and creativity in order to inspire high school students to see probability in their lives around them and be comfortable analyzing it.

Rationale: In my brief tenure as a math teacher in the School District of Philadelphia I have noticed that students get confounded when approached with slightly more complex situations of probability. They find it easy to consider flipping a single coin or rolling one or two dice, yet if asked about the probability in their favorite games they are unable to make the connections necessary to calculate the answer. Moreover, they do not understand how to apply these principles to their daily lives – they are barraged with information that could help them make better choices yet they ignore it because they do not analyze it like data.

In analyzing probability scenarios in math classes I have also noted that many students have a difficult time explaining their conclusions in terms of fractions, decimals, and percents. Instead they focus on abstract terms that do not denote the measurement of probability in different situations and may sometimes convey a lack of understanding of probability in general. Students need to gain an understanding of fractions, decimals, and percents and this unit may provide the context for them to hold onto something concrete and explainable.

I think it is important that students understand the basics of probability and fractions, decimals, and percents not just for tests like the Benchmark and Predictives but also for their own lives. I will endeavor to create a unit that will entice students to become more invested and interested in mathematics by showcasing aspects of their own communities. My limited experience will drive me to explore topics like the lottery and the new Sugar House casino that recently opened in Philadelphia. I believe that using these real-life examples will provide concrete context for students to refer back to in all aspects of their lives so that they can remember important mathematical rules regarding fractions, decimals, and percents.

In preparation for this unit, I have researched various different types of probability games in the United States and abroad. I think it is important to gain an understanding of different cultures and their use of math for the students to connect mathematics to a larger world context and see the connections between things foreign and domestic. I use computerized versions of some of these games so that we can play them during class without a large setup and to infuse the use of technology into the curriculum. I have also researched various suggestions made by other educators as well as the Teach For America bank of lesson plans.

Objectives: This unit is intended for students in Grade 9 who are studying Algebra 1. They spend 56 minutes of each day in this class. The class is structured with a beginning activity (a "PromPT"), an overview of the objectives for the day, an agenda, any direct instruction necessary, with group or individual practice, or a lab activity of some kind, and an exit question to assess comprehension.

Over the course of the unit students will be expected to experiment with various types of games of chance. The teacher will provide some games and others will be provided or created by the students. Some days will be focused on individual skills used to analyze probability (ex. Converting between decimals, fractions, and percents) while other days will be used to create the games being analyzed or listening to speakers who may be brought in to discuss chance in their jobs.

The major end goal will be to create and analyze a game based on probability. Students will work in groups to achieve this and will be able to present and explain the math behind their game to their classmates.

The overarching objectives for the unit will include the following:

- Convert between fractions, decimals, and percents
- Determine or calculate the number of possible outcomes for a particular event
- Calculate the experimental or theoretical probability of the outcomes of a simple or compound event
- Explain the difference between replacement and non-replacement situations and calculate the associated probabilities accordingly

Strategies: This unit will include various craft activities to connect students' creativity with their math skill by designing probabilistic games. It will also include analysis of these games as well as other, better known games that are brought in. Students will then play and analyze several simple probability games in order to gain an understanding of the mathematics behind spinners, dice, cards, etc.

Specific tools to be used include the physical games mentioned above as well as digital versions for ease of duplicity and transport. Various websites and java applets will provide tools that students can use to experiment with probability on their own and in small groups. These tools will be useful in discussing the aspects of the probabilistic games they create as well as calculating the probabilities of certain outcomes therein.

Toward the end of the unit students will analyze real-life situations that involve probability (ex. Genetic or disease testing). This will galvanize the connection between probability and the real world.

Activities: Each student will perform the following tasks in order to demonstrate his or her knowledge of the subject area:
- Create a playable game
- Analyze the game to determine the possible outcomes and probabilities thereof
- Research a probability game from another culture/country and present on its background and probability
- Create a tutorial video used to explain the relationships between decimals, fractions, and percents.

Essential Questions:
- What makes a game fair or unfair?
- What is probability?
- How do you measure the likelihood of an event?
- How do you represent the likelihood of an event?

Daily Structure: Each day will have an objective on which to focus. To begin the class, students will complete an activity to prompt their thoughts (**Prom**ethean **P**ractice assignmen**T**). After this activity they will engage in an interactive discussion, lecture, lab, or other process to introduce them to some form of new material if necessary. The main activity might be a continuation of the introduction or a completely new approach. Students will be assessed on their knowledge at the end of class through an "Exit Question." This might be a writing exercise, verbal exercise, presentation, movement response, or something to demonstrate the level at which the content was mastered.

Class will also include a Math Journal for students to record notes, questions, thoughts, etc. This Journal will be checked every 2 weeks for maintenance.

Objective: SWBAT (Students will be able to) explain experimental and theoretical probability using fractions, decimals and percents in real-life situations

PromPT: Students write down a 3-sentence response to the question "Is life fair?" on a piece of paper.

Intro to New Material: After reading through the objective with students, ask them to make groups of 3-4 and have each group agree on an answer with an explanation. Share explanations as a group.

Main Activity: Put trashcan in front of room and ask students to come up (one from each group) to see what is the probability of them shooting a small ball into the basket. Calculate this experimental probability using fractions. Introduce math journal and have students put answers to certain questions in it. Then discuss concept of a probability scale and have students come up with examples of impossible situations to put by the "0" part of the scale. Then talk about certain situations and put by the "1" part of the scale. Then talk about "1/2" part of scale.

Exit Question: Math journal question – create your own graphic organizer for a probability scale. Come up with your own examples for impossible, certain, and half way.

Differentiation Possibilities: Provide graphic organizer template for student to use in journal.

Day 2

Objective: SWBAT explain experimental and theoretical probability using fractions, decimals and percents in real-life situations

PromPT: Organize the following probabilities in order on the scale in your math journal: 1/8, 3/8, ½, 7/9, 2/3.

Intro to New Material: Explain how to represent the choices made in the Mystery Pasta Game to follow (put the total number of items on the bottom of fraction, the number of

items that you are focusing on goes on top). Model to students how the Mystery Pasta Game is played (each bag has various amounts of various different kinds of pasta; students pick out pasta pieces at random and record what they received; after each removal the pasta is replaced).

Main Activity: Students are given three bags of different combinations of pasta. They must record what they remove from the bags on a template to help record the probability of getting a certain kind of pasta. The bags will be labeled and correct amounts in each bag will be displayed on the board. The catch: the amounts in each bag are not labeled so the students have to figure out which bag has which amount without taking them all out.

Exit Question: If a bag of M&Ms consists of 4 reds, 3 blues, 2 greens, and 5 yellows, what is the probability of removing a green one?

Differentiation Possibilities: Partner struggling students with faster students for better results.

Day 3

Objective: SWBAT explain a sequence of probabilistic outcomes by creating a simple probability tree

PromPT: Math Journal - What makes a game fair?

Intro to New Material: Ask various students to play against me in Rock, Paper, Scissors and record wins/losses on the board. Determine if I am really good or not based on Win/Loss record.

Main Activity: Divide the class into pairs (player A and player B) and have them play the game 15 times. Record results in different colors (red for if player A wins, blue for if players B wins, green for if it is a tie). Compare the results. Ask the class: *Is this game fair?* Explain that this means equal chance of winning for all players. Ask students to explain why they think it is fair. Try to elicit from students that it is fair because each student has an equally likely or equal chance of winning (50% or 1/2). Introduce concept of tree diagram as a visual tool; have students attempt to make their own trees for this game.

Introduce students to a tree diagram as a visual tool for keeping track of the possible outcomes of this game: This is known as a probability tree. To address the Content question, "*What is the difference between experimental and theoretical probability,*" compare this mathematical model with what happened when students played the game (theoretical vs. experimental probability).

Exit Question: Imagine a person is flipping a coin (with a heads side, and a tails side) twice. Make a probability tree for this scenario.

Differentiation possibilities: Have a tree diagram prepared for this situation. Students can fill it in following the patterns we discuss.

Day 4

Objective: SWBAT explain the difference between theoretical and experimental probability by making a probability tree

PromPT: Word association with the words "theoretical" and "experimental" as they relate to probability. Have students write the words on the board as they walk in.

Intro to New Material: Put a simple tree diagram on the board and have students "tell a story" about what it could mean. Have them fill in the blanks (like the probabilities for each branch and what each branch means).

Main Activity: Divide up students into 4 different lab stations. Have them each play with the simple probability game (spinner with 1-5, coin, one die with different outcomes for each number, one die with different outcomes for 1/4, 2/5, and 3/6). Have students make trees for each game and experiment to see if the experimental and theoretical probabilities match.

Exit Question: Explain the difference between theoretical and experimental probability by using an example from one of the lab stations today.

Differentiation Possibilities: Provide a tree template for one of the simpler stations and have student fill in the blanks for probabilities and what each branch means.

Day 5

Objective: SWBAT research and describe a simple probability game from outside the US

PromPT: Students walk in and are immediately given a penny each. They must toss the penny and record heads or tails 10 times. They are asked to determine if their results are better or worse than the theoretical probability of ½ for each. This record goes in their Math Journals.

Intro to New Material: Show pictures of boards/cards of foreign games on the board (ex. Lu-Lu from Hawaii, Igba-Ita from Nigeria, etc.) and have students use Notice/Wonder method for them. A student can lead this activity where s/he calls on others to comment on what they notice and wonder. This will lead to larger conversation about similarities/differences to games known in the United States.

Main Activity: Students are provided with a host of options of foreign probability games to research. In pairs of two the students will research a brief history of the game and the rules. They will be tasked with writing 3 paragraphs about where/how the game developed and how it is played.

Exit Question: What is one interesting fact you discovered about the game you researched?

Differentiation Possibilities: Provide student with a specific game to research as well as websites/books about it.

Day 6

Objective: SWBAT research and describe a simple probability game from outside the US

PromPT: Provide rules for a foreign game and have students play for 5 minutes at their tables. Ask for thoughts on game.

Intro to New Material: n/a

Main Activity: Students complete their research and writing today. Randomly pick pairs to share information they found for extra credit on their assignment.

Exit Question: What is a rule for a new game you heard about today?

Differentiation Possibilities: Provide extra time outside of class to complete research assignment.

Day 7

Objective: SWBAT calculate conditional probabilities using examples of real-life games

PromPT: What are the possible outcomes of rolling two dice in a row? Make a probability tree for this situation.

Intro to New Material: Define conditional probability as the likelihood of Event A occurring given Event B has already occurred.

Main Activity: A round robin of casino games to analyze. At each station there is a card or dice game that is usually at casinos. At each station there are questions regarding the play of the game. Potential games include Craps (what are the chances of a 6 being rolled if a 7 has already been rolled?), Black Jack (If your first two cards were the jack of spades and 8 of clubs, what are the odds of pulling a black card next?), Slot Machines (If there are 3 spinners each having 5 pictures of cherries and 20 pictures of other food, what are the chances of getting at least 2 cherries?), etc.

Exit Question: Calculate the conditional probability of drawing a six from a deck of cards if one seven has already been drawn.

Differentiation Possibilities: Student will focus on fewer games instead of completing the entire round robin.

Day 8

Objective: SWBAT calculate conditional probabilities using examples of real-life games

PromPT: Put up a probability tree and have students analyze it to determine what the conditional probabilities would be given certain situations.

Intro to New Material: n/a

Main Activity: Provide students with probability trees representing different real-life game situations. Have pairs of students come up with a story to explain the various different options that are possible within these trees. Write down stories and have them read the stories to the class. Other pairs have to guess what the probability tree looks like and try to recreate it.

Exit Question: What did you find easy/difficult about the activity today?

Differentiation Possibilities: Provide some example sentences to use for groups struggling with this.

Day 9

Objective: SWBAT apply probability calculations to real-life situations and make choices

PromPT: Put up probability tree of a three-pictured slot machine and have students determine probabilities of getting money at the end.

Intro to New Material: Show video on situation where a sample of fish in a pond is used to determine what the proportions of each type of fish is in the pond.

Main Activity: Conduct lab activity mirrored on pasta activity of Day 2, but without telling the amounts in each bag. This time, have students remove a sample from the bag and draw on the sample with markers to "tag" the pasta. Place it back in the bag and take another sample out. Use the proportion of "tagged" pasta to calculate how many of each type is in the bag.

Exit Question: What is an example of using probability in real-life for you?

Differentiation Possibilities: Provide a template with specific instructions on which to calculate proportions.

Day 10

Objective: SWBAT apply probability calculations to real-life situations and make choices

PromPT: Write 1 sentence of how you could use probability in real life on the board.

Intro to New Material: n/a

Main Activity: Provide students with 15 minutes to make a one-slide, one-minute powerpoint presentation on using probability in their daily lives. The students will work in pairs and have to run the idea by the teacher first to ensure no duplications. They will present in the second half of class and be sure to mention some kind of calculation of probability in their examples.

Exit Question: What is one way you learned to use probability in real life today?

Differentiation Possibilities: Allow students to present to me outside of class if they are nervous. Provide a few suggestions of what to research quickly for this.

Day 11

Objective: SWBAT work in groups and use fractions, percents, and decimals to create and analyze their own probability game

PromPT: Convert among decimal, fraction, percent – 1/3, 0.25, 35%, 0.75, 3/5, 40%

Intro to New Material: n/a

Main Activity: Provide guidelines to students on what kind of game they can produce (dice-, card-, or spinner-based games are appropriate). There must be at least 4 rules relating to probabilistic outcomes for the game. Students will prepare a presentation of this game as well to the class, which will act as a panel to approve or not approve the game for a toy company.

Exit Question: What type of game did you pick (dice, card, spinner) and why?

Differentiation Possibilities: Struggling students will be provided with a template of a spinner to use and potential outcomes in a game that are easier to understand.

Day 12

Objective: SWBAT work in groups and use fractions, percents, and decimals to create and analyze their own probability game

PromPT:

Intro to New Material: n/a

Main Activity: Provide guidelines to students on what kind of game they can produce (dice-, card-, or spinner-based games are appropriate). There must be at least 4 rules relating to probabilistic outcomes for the game. Students will prepare a presentation of this game as well to the class, which will act as a panel to approve or not approve the game for a toy company.

Exit Question: What rules did you come up with today for your games?

Differentiation Possibilities: Struggling students will be given some suggested questions that will guide rule-making for the games.

Day 13

Objective: SWBAT work in groups and use fractions, percents, and decimals to create and analyze their own probability game

PromPT: Draw a card from the deck to determine order of presentations (lower card means earlier presentation)

Intro to New Material: n/a

Main Activity: Students will present their games to class in 2-minute maximum presentations. Classmates will grade each other's presentations using a pre-defined rubric. Each class will vote on the top 3 games present those to show to a small panel of

adults in the casino/game industry the next day. Students will then play games in a carnival-like atmosphere with food/drink (brought in by students and some from teacher).

Exit Question: What was your favorite game today and why?

Differentiation Possibilities: Struggling students can present game to teacher separately.

Day 14

Objective: SWBAT interpret and question speakers on concepts of probability in games

PromPT: Write down one question you have for the speakers on a piece of paper. The question must relate to probability and/or games in some way.

Intro to New Material: n/a

Main Activity: Speakers will introduce themselves and talk about how probability is used in their careers. Speakers should be from a variety of backgrounds including game-maker/designer, representative from local casino, and/or a non-game-based career.

Exit Question: What is one thing you learned from the speakers today?

Differentiation Possibilities: n/a

Annotated Bibliography:

Reading List

Mathematics Teacher. NCTM publication.
        Monthly journal providing articles that examine best practices in mathematics education and suggest methodologies for implementing them.

Zaslavsky, Claudia. *Math Games & Activities from Around the World.* Chicago, IL: Chicago Review Press, 1998.
        Provides various descriptions of games of chance played around the world from cultural backgrounds of all kinds. Divides games into similar types.

Teacher Resources

Algebra: The Supplement. < http://algebra.mrmeyer.com/>. Website on Algebra 1 topics.
        Dan Meyer, math teacher from California, provides Algebra 1 activities and resources to be used by teachers around the world.

Probability – from Wolfram MathWorld.
<http://mathworld.wolfram.com/topics/Probability.html>. Website on Probability topics.
        Definitions and examples of probability within mathematics and the real world.

Web Links – Data Analysis & Probability.
< http://illuminations.nctm.org/WebResourceList.aspx?Ref=2&Std=4&Grd=0>. Website with links to various probability activities, lesson plans, and resources.
        National Council of Teachers of Mathematics provides activities, lesson plans, and resources from which to teach probability to students.

Student Resources

Probability explorations. < http://illuminations.nctm.org/LessonDetail.aspx?id=U190>. Website with various activities to explore probability.
        Java applets used by students to explore concepts of probability while creating grounding experiences in something manipulated.

Adjustable Spinner. < http://illuminations.nctm.org/activitydetail.aspx?ID=79 >. Website with adjustable spinner for probability activities.

       Java applets used by students to complete probabilistic activities analyzing various situations.

Appendix

Standards: The Core Curriculum of the School District of Philadelphia is aligned with the Pennsylvania Academic Standards for Mathematics. These standards include instruction on the following topics: Mathematical Problem Solving & Communication and Probability and Predictions. Many life concepts can fall into these categories so it is important to maintain connection with real-world examples while teaching them. Students will focus on problem solving techniques as they relate to probability in order to extrapolate probabilistic information from the world around them.

- 2.5.A1.A. Develop a plan to analyze a problem, identify the information needed to solve the problem, carry out the plan, check whether an answer makes sense, and explain how the problem was solved in grade appropriate contexts.

- 2.5.A1.B. Use symbols, mathematical terminology, standard notation, mathematical rules, graphing, and other types of mathematical representations to communicate observations, predictions, concepts, procedures, generalizations, ideas, and results.

- 2.7.A1.A. Calculate probabilities for independent, dependent, or compound events.