

Is This Fair?

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Overview:

This unit is designed to help students develop a better understanding of probability and chance. The unit focuses on the concepts of fair and unfair and logic as they relate to the rules of games. The unit will explore the skills of counting possible outcomes, determining if the outcomes are equally likely or not, calculating the probability of each outcome, and finally using probability to determine if the rules are fair and make sense.

This unit is designed for a self-contained third grade classroom that is following the School District of Philadelphia's Core Curriculum with *Everyday Mathematics*. The unit is intended to reinforce the Pennsylvania state mathematics probability and prediction standards already covered in Kindergarten through second grade and provide the students with a more solid base of understanding for the standards that will be covered in third grade and beyond.

Probability is a concept that the *Everyday Mathematics* curriculum touches on here and there throughout the year. It's not a concept however, that the curriculum spends a significant amount of time on. As a result the students have a difficult time connecting the different probability concepts they are taught. They also have a difficult time connecting the idea of probability to their lives. This unit will cover the concept of probability once a week for six to eight weeks, which will give the students time to master and connect the different probability concepts and skills taught to them previously and also help them to master the probability concepts and skills that will be taught to them in the future.

The unit will teach the students strategies they can use to count possible outcomes of an experiment.

Rationale:

It can often be difficult to connect third grade mathematics to the real lives of third graders from lower socioeconomic backgrounds. The concept of fair versus unfair is very real and important to third graders however. Students often get upset and declare that “It’s not fair!” when they lose at something.

This unit will take place within the confines of the Core Curriculum. Lessons in this unit will be taught once a week on Fridays for six to eight weeks. Most Fridays in the Core Curriculum are used for playing Everyday Mathematics games and review. The lessons in this are not meant to replace the math games and review. The lessons will incorporate the math games into the unit by having the students analyze the rules of the games to determine A) if the games are fair and B) if the rules make sense. The lessons will also save time throughout the year when the Friday review would be about probability. Students will construct an understanding of basic probability concepts in this unit, so less time is required for formal review of them.

Mathematical Background:

The following probability terms are important to know and understand to teach this unit; sample space, outcomes, event, experiment, and probability. A sample space is the set of things that could happen. Outcomes are the basic things that could happen i.e. the elements of the sample space. An event is a set or collection of outcomes. The probability of an event is the chance that the event will happen expressed as a percentage or a number between 0 and 1.

Objectives:

This unit seeks to give students a thorough understanding of how to determine if a game or activity is fair and whether the rules of the game are logical. The unit will examine fairness through the use of probability.

The unit will break down the understanding of probability and fairness into three basic questions: How many possible outcomes are there for the experiment of playing one’s turn? Are all of the outcomes equally likely? And if not, does the unequal probability give an advantage to one of the players?

The students will understand what an experiment and an outcome are. The experiments in this unit will be playing one’s turn. They will learn to use the Four-Step Method and a table or an organized list (see Strategies below) to count the possible outcomes of an experiment.

Strategies:

The students will be taught the following strategies during this unit.

The Four-Step Method (The Problem Solver 3)

- 1) Find Out
- 2) Choose A Strategy
- 3) Solve It
- 4) Look Back

Find Out: The first step of the Four-Step Method is to understand what the problem is about. The students will have to comprehend the words and phrases used in the problem. They will have to be able to identify the question in the problem.

Choose a Strategy: The second step of the Four-Step Method is choosing an appropriate strategy that will help you solve the problem. The students should realize that there is usually more than one way to solve a problem however certain strategies are more helpful for particular problems.

Solve It: The third step of the Four-Step Method is solving the problem. The students will need to be able to use their strategies to work through the problem.

Look Back: The fourth step of the Four-Step Method is to check your work. The students should reread the problem and make sure their answer makes sense.

Use or Make a Table (The Problem Solver 3)

One example of using the four-step method is making a table. A table is a way to organize data. Tables help students “keep track of data, spot missing data, and identify data that is asked for in a problem.” Making a table doesn’t directly relate to probability.

Example:

Problem 35 from The Problem Solver 3: The guards of Clock Castle open the two gates at the same time every hour to let visitors go into the castle. The guards keep track of how many people they let in through their gates. This is what they wrote today.

Time	8:00	9:00	10:00	11:00	12:00	1:00	2:00	3:00
Visitors through the King’s Gate	1	5	9	13	17			
Visitors through	1	4	9	12	17			

the Queen's Gate									
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The guards kept letting visitors go into the castle in the same way all day. How many visitors went through Queen's Gate at the time that 33 visitors went through King's Gate?

Solution: 33 visitors

Time	8:00	9:00	10:00	11:00	12:00	1:00	2:00	3:00	4:00
Visitors through the King's Gate	1	5	9	13	17	21	25	29	33
Visitors through the Queen's Gate	1	4	9	12	17	20	25	28	33

Make an Organized List

An organized list helps students to keep their ideas about a problem organized. It is easier to look back and review the work that they already have done.

Example:

Problem 3 from The Problem Solver 3: The sign says: JULIO'S MAGIC SHOW. Julio is behind the curtain, getting ready for the show. He has to make up his mind what to put on. He will wear a cape or a coat. On his head he will wear a bright red wig or a hat. Then he will put on his black boots or his shoes with the pointy toes. What are the 8 different outfits Julio could wear for his magic show?

Solution:

	Piece for Shoulders	Piece for Head	Pieces for Feet
Outfit 1	coat	wig	boots
Outfit 2	coat	wig	shoes
Outfit 3	coat	hat	boots
Outfit 4	coat	hat	shoes
Outfit 5	cape	wig	boots
Outfit 6	cape	wig	shoes
Outfit 7	cape	hat	boots
Outfit 8	cape	hat	shoes

Classroom Activities:

Lesson Plan #1 (Counting)

Duration: Two days (about 45 minutes per day)

Objectives:

The students will be able to count the number of possible outcomes of an experiment. They will be able to organize the outcomes in an organized list or a table. Students will understand that making an organized list or table will help them keep track of outcomes, spot missing outcomes and identify outcomes. They will also understand that organized lists and tables make it easier to discover patterns.

PA Mathematics Standards:

2.7C. List or graph the possible results of an experiment.

Materials:

The Problem Solver 3: Problem # 3

A spinner with four equal sections for each group

A regular six-sided die for each group

Various-sided dice

Plans: (day 1)

1. Warm Up (5 minutes)
 - a. The teacher takes out a rectangular prism.
 - b. The teacher reviews what a face is in regards to three-dimensional shapes.
 - c. The teacher models counting the faces of the rectangular prism.
 - d. The teacher passes out various three-dimensional shapes.
 - e. The students count the number of faces on their shape.
2. Introduction (5 minutes)
 - a. The teacher refers to the objectives on the board.
 - b. The teacher reads the mastery objective to the students.
 - i. Students will be able to count the number of possible outcomes of an event.
 - c. The teacher goes over what the objective means.
3. Model (10 minutes)
 - a. The teacher passes out six-sided dice
 - b. The teacher thinks out loud about the different possible outcomes when rolling the die one time.

- c. The teacher makes a list of the outcomes on the board making sure not to put the outcomes in order.
 - d. The teacher thinks out loud about how it would be easier to keep track of the results if they made an organized list.
4. Guided Practice (10 minutes)
- a. The teacher passes out various sided dice.
 - b. The teacher instructs the students to make an organized list of the possible outcomes of rolling their die one time.
 - c. The students should work in groups.
 - d. The teacher goes over the students results
5. Independent Practice (10 minutes)
- a. The teacher passes out spinners to each group.
 - b. The teacher instructs the students to make an organized list of the possible outcomes of spinning the spinner one time.
 - c. The teacher should go over the results.
6. Wrap Up (5 minutes)
- a. The teacher reviews the vocabulary of organized list, event, and outcome.

Plans: (day 2)

7. Warm Up (5 minutes)
- a. The teacher takes out a six-sided die.
 - b. The teacher will ask what the possible outcomes are.
 - c. The teacher will make an organized list on the board of the outcomes.
8. Introduction (5 minutes)
- a. The teacher takes out a second different colored die.
 - b. The teacher will ask the students to predict how many different possible outcomes they think there will be if they rolled both dice. (Make sure to use the term outcomes.)
 - c. The teacher will record the predictions on the board.
9. Model (5 minutes)
- a. The teacher will think out loud as they model how to start a table to count the possible outcomes of rolling two six-sided dice.
10. Guided Practice (15 minutes)
- a. The teacher passes out grid paper to the students.
 - b. The teacher helps them set up a grid like their table on the board.
 - c. The teacher and the students work together to fill in the table.

- d. The teacher passes out *The Problem Solver 3: Problem # 3*
 - e. The teacher helps the students set up their organized lists.
11. Independent Practice (10 minutes)
- a. The students finish the organized list for *The Problem Solver 3: Problem # 3*
12. Wrap Up (5 minutes)
- a. The teacher reviews the vocabulary of organized list, experiment, and outcome.

Lesson Plan #2 (Equally Likely?)

Duration: Two days (about 45 minutes per day)

Objectives:

The students will be able to count the number of possible outcomes of an experiment. They will be able to determine if all of the possible outcomes are equally likely or not.

PA Mathematics Standards:

2.7.A. Predict and measure the likelihood of an outcomes and recognize that the results of an experiment may not match predicted outcomes.

Materials:

A spinner with four equal sections for each group

A regular six-sided die for each group

Various - spinners

Quarters (real or from the Everyday Mathematics kit)

Worksheet 2A

Plans: (day 1)

1. Warm Up (5 minutes)

- a. Take out a quarter and ask the students to write down the possible outcomes if you flip it. (Be sure to use the word *outcomes*.)
- b. Take out a regular six-sided die and ask the students to write down the possible outcomes if you roll it.
- c. Take out a spinner with four equal sections and ask the students to write down the possible outcomes if you spin it.
- d. Go over the students' answers.

2. Introduction (10 minutes)

- a. The teacher should review what the previous lesson. Make sure the students remember the definition of the terms *an outcome* and *outcome*.
- b. The teacher should take out the materials from the warm up.

- c. Ask the students what the experiment is when you flip the coin.
- d. Ask the students if they think that there is an equal chance of getting heads or tails if you flip the quarter. (Make sure to use the term *equal chance*.)
- e. Make a tally cart on the board of their responses.
- f. Ask the students what the experiment is when you roll the die.
- g. Ask the students if they think that there is an equal chance of getting a 1, 2, 3, 4, 5, or 6 when you roll the die.
- h. Make a tally chart on the board of their responses.
- i. Ask the students what the experiment is when you spin the spinner.
- j. Ask the students if they think there is an equal chance of getting each section of the spinner when you spin it.
- k. Ask the students why they think yes or no for each experiment. (Make sure to use the term *experiment*.)

3. Model (10 minutes)

- a. The teacher should do a think aloud.
- b. Look at the quarter. Explain that you know that there is an equal chance of getting heads or tails because the two sides of the coin are the same size.
- c. Look at the die. Explain that you know that there is an equal chance of getting a 1,2,3,4,5, or 6 because the six sides of the die are all the same size.

4. Guided Practice (10 minutes)

- a. Take two various spinners. One that has each sections and one that has unequal sections.
- b. Start with the spinner with equal sections.
- c. Have the students come up front and spin the spinner or pass the spinner around the room.
- d. Keep a tally on the board of how often the spinner lands on each section.
- e. Make sure to do at least 50 spins. (The spinner should land on each section *about* the same number of times. Please note that the number of times that the spinner lands on each section will not be exactly equal.)
- f. Think aloud about how the sections on the spinner are equal in size and that each outcome came up *about* the same number of times. (Make sure to use the word *outcome*.)
- g. Take out the spinner with unequal sized sections. Ask the students what they notice about this spinner compared with the last spinner.
- h. Have the students come up front and spin the spinner or pass the spinner around the room.
- i. Keep a tally on the board of how often the spinner lands on each section.
- j. Make sure to do at least 50 spins. (The spinner should land on largest section the most times and the smallest section the least number of times. Please note that the number of times that the spinner lands on each section will not be exactly proportional to their sizes.)

- k. Think aloud about how the sections on the spinner are unequal in size. Talk about how the largest section was the most likely outcome and the smallest section was the least likely outcome. (Make sure to use the word outcome.)
5. Independent Practice (10 minutes)
- a. Put the students in small groups or partners.
 - b. Pass out various spinners and worksheet 2A.
 - c. Have the class use the spinners to complete the worksheet.
6. Wrap Up (5 minutes)
- a. Have groups share their results.
 - b. Make sure to point out that groups with spinners with equal sized sections had each outcome come up *about* the same number of times and groups with spinners with unequal sized sections probably had the largest section come up most often and the smallest section come up least often.
 - c. Make sure to point out that those groups with spinners with equal sized sections probably did not see each outcome come up exactly the same number of times and that it is possible, although not likely, for a smaller section to come up more often than a larger section.

Plans: (day 2)

1. Warm Up (5 minutes)

- a. Take out a spinner with unequal sized sections. (A large demonstration spinner if you have one. You could also use an overhead projector, a smart board, or simply draw a spinner on the board.)
- b. Tell the students that you're going to play a game. This is a game for two people. In this game each player gets to pick sections on a spinner. The players take turns spinning the spinner. If the spinner lands on your section when you spin it, you get a point. If it lands on your opponent's section, nobody gets a point.
- c. Ask the students which sections they would pick first and why.
- d. When the students answer, get them to use appropriate mathematical terms like experiment, outcome, more likely or less likely.

2. Introduction (10 minutes)

- a. Review any terms or concepts from the previous lesson that didn't come up in the introduction.
- b. Tell the students that they are going to design a spinner.
- c. Their assignment is to design a spinner for the game we just played. Their spinner should be unfair. It should be unfair in that whoever gets to pick a section first will be more likely to win. However, they should try to make the game seem as fair as possible.

3. Model (10 minutes)
 - a. Show the students the blank spinner worksheets.
 - b. Demonstrate how to use a ruler and protractor (Only use a protractor if they are already familiar with how to use one.) to divide the spinner into sections.
 - c. Demonstrate how to use a pencil and paperclip to make and spin the spinner. (See appendix.)
4. Guided Practice (10 minutes)
 - a. Pass out materials.
 - b. Circulate while the students work on their spinners.
5. Independent Practice (10 minutes)
 - a. Have the students pick partners to play their game with.
 - b. Each group will play use both spinners to play. The student who designed the spinner will get to pick first. Have the students play for long enough to spinner about 50 times.
6. Wrap Up (5 minutes)
 - a. Have the students report out.
 - b. Ask the students if they won on the spinner they designed.
 - c. Ask the students if the spinner they didn't design looked pretty fair. (Use and encourage the students to use the terms; fair, unfair, more likely, less likely, and outcome.)
 - d. Ask why they spinners were fair or unfair.

Lesson Plan #3

Duration: Two days (about 45 minutes per day)

Objectives:

- a. The students will be able to analyze the rules of an *Everyday Mathematics* game.

PA Mathematics Standards:

2.7A. Predict and measure the likelihood of experiments and recognize that the results of an experiment may not match predicted outcomes.

2.7C. List or graph the possible results of an experiment.

2.7D. Analyze data using the concepts of largest, smallest, most often, least often and middle.

Materials:

Baseball Multiplication boards (Many of the *Everyday Mathematics* games will work.

Use a game that the class is familiar with.)

Dice (two different colors)

Counters

Plans: (day 1)

1. Warm Up (5 minutes)

- a. Have the students roll dice and multiply the numbers that they get.

2. Introduction (10 minutes)

- a. Review the rules of Baseball Multiplication

3. Guided Practice (10 minutes)

- a. Ask the students to help name the sample space for rolling two dice.
- b. Draw it on the board.
- c. Ask the students what the most desirable numbers to score are.
- d. Think aloud that the most desirable number to roll is 36. There is only 1 way to roll a 36 so the probability of rolling a 36 is $1/36$.
- e. Ask the students why they think the maker of the game made 36 the number to roll to get a 36.

4. Independent Practice (10 minutes)

- a. Have the students make a chart or an organized list of the probability of rolling a triple, a double, a single, and striking out.

5. Wrap Up (5 minutes)

- a. Ask the students if they noticed any patterns.
- b. Tell them that next time we will examine the patterns more closely.

Plans: (day 2)

1. Warm Up (5 minutes)

- a. Have the students roll dice and multiply the numbers that they get.

2. Introduction (10 minutes)

- a. Redraw the sample space for rolling two dice on the board.

3. Guided Practice (20 minutes)

- a. Go over the probabilities for each event. (Home run, triple, double, single, and out)
- b. The probability for getting a home run is $1/36$.
- c. The probability for getting a triple is $3/36$.
- d. The probability for getting a double is $7/36$.
- e. The probability for getting a single is $15/36$.
- f. The probability for getting an out is $10/36$.

5. Independent Practice (10 minutes)

- a. Let the students play Baseball Multiplication.

- b. Tell them to look for anything that makes the game unfair to the player who is up first or the player who is pitching first.
6. Wrap Up (5 minutes)
- a. Ask the students if they think the game is fair.
 - b. Ask the students if the rules make sense and are logical.

Lesson Plan #4

Duration: Two days (about 45 - 65 minutes per day)

Objectives:

Students will be able to design a fair and logical game.

PA Mathematics Standards:

2.7A. Predict and measure the likelihood of experiments and recognize that the results of an experiment may not match predicted outcomes.

2.7B. Design a fair and an unfair spinner.

2.7D. Analyze data using the concepts of largest, smallest, most often, least often and middle.

Materials:

Blank spinners

Dice

Cards

Poster board

Scissors

Glue

Crayons

Plans: (day 1)

1. Warm Up (5 minutes)

- a. Review some of the *Everyday Mathematics* games that we have played.

2. Introduction (10 minutes)

- a. Explain to the students that they are going to be making their own math games.
- b. Pass out Math Game Making Instruction worksheet.
- c. Go over the instructions.

3. Work Time (40 minutes)

- a. Circulate and assist student groups as they work.

5. Wrap Up (10 minutes)

- a. Have the groups share what they have done so far.
- b. Have other groups offer suggestions.

Plans: (day 2)

1. Introduction (10 minutes)

- a. Review the Game Making Instruction worksheet.
- b. Ask if there are any questions.

2. Work Time (40 minutes)

- a. Circulate and assist student groups as they work.

3. Wrap Up (5 minutes)

- a. Have each group give a one minute overview of their game.

Annotated Bibliography / Resources:

The Problem Solver 3: Activities for Learning Problem-Solving Strategies

School District of Philadelphia Core Curriculum Grade 3 Mathematics

Everyday Mathematics Teachers Edition

Websites

www.EdHelper.com/probability.html

www.free-training-tutorial.com/probability-games.html

Appendix / Content Standards:

Pennsylvania State Mathematic Standards

2.7A. Predict and measure the likelihood of experiments and recognize that the results of an experiment may not match predicted outcomes.

2.7B. Design a fair and an unfair spinner.

2.7C. List or graph the possible results of an experiment.

2.7D. Analyze data using the concepts of largest, smallest, most often, least often and middle.

Worksheets

2A, Blank Spinner, Math Game Making Instructions

1) My spinner had _____ different sections.

2) When I spin the spinner there are _____ possible outcomes.

3) The possible outcomes are _____
_____.

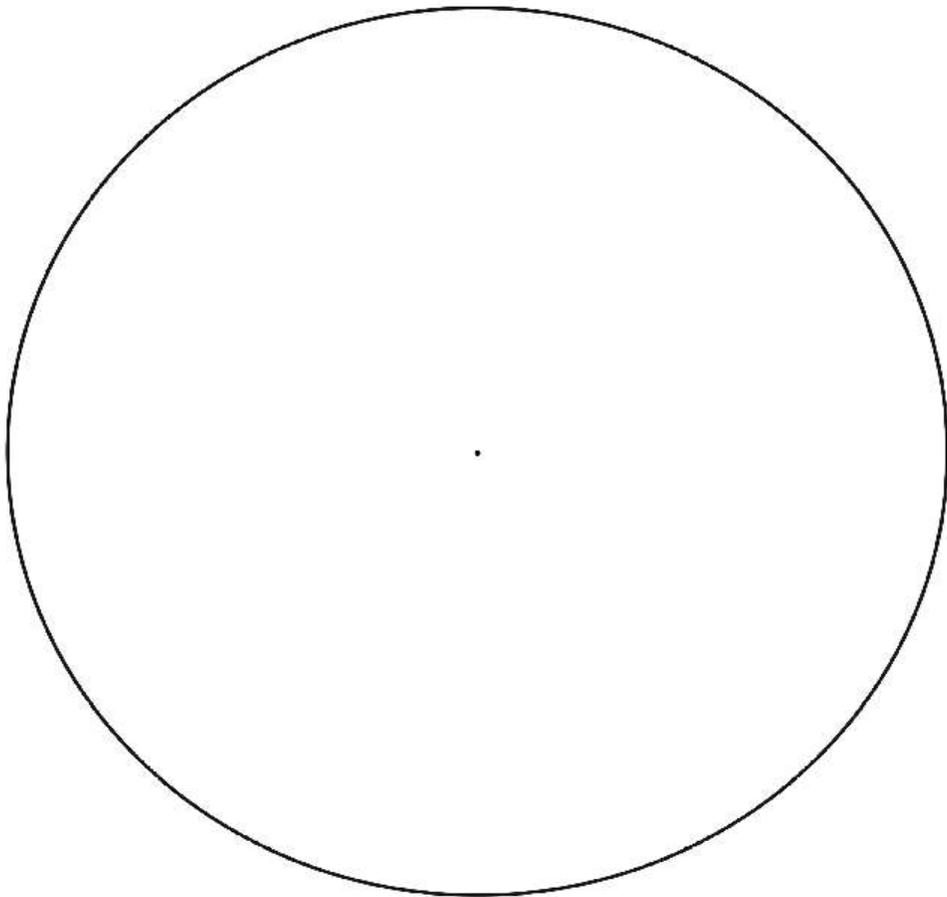
4) The sections are all the same sizes or different sizes.
Circle one.

Section	Number of times

Total _____

5) Are your results what you expected? Why or why not?

Blank Spinner Template



Directions for making your own spinner

Materials

Pencil

Ruler

Paperclip

Protractor (optional)

- 1) Use the protractor (optional) and a ruler to divide the circle into sections.
- 2) Label or name each section.
- 3) Place the paperclip on the circle so that the dot in the center of the circle is inside of the paperclip.
- 4) Hold the pencil so the point is in the middle of the paperclip and right on the dot in the center of the circle.
- 5) Flick the paper clip to spin it around.

Rules for Baseball Multiplication

- Players take turns being the “pitcher” and the “batter”.
- Players choose a scoring chart and the correct number of dice to use for the game.
- The pitcher rolls the game.
- The batter finds the product of the numbers, and checks the scoring chart. The batter moves the hitting counter the number of bases shown in the scoring chart. All other counters on the field also move ahead.
- Some products are outs. Players check the scoring chart for outs.
- Incorrect answers are strikes. After a strike, another “pitch” (dice roll) is thrown. Three strikes makes an out.
- When a team gets 3 outs, their turn is over.

Scoring Chart for Baseball Multiplication

36 = Home run

25 to 35 = Triple

16 – 24 = Single

5 or less = Out

Math Game Making Instructions

Today you are going to get to make your very own math game. Your game must show your understanding of the probability concepts we have been going over in class.

- 1) Your game must be fair. That means that all players have an equal chance of winning the game no matter when their turn is.
- 2) Your game can use spinners, dice, or coins.
- 3) Your game should be fun to play.

