Winner Take All?
Exploring Probability through Games of Chance

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Overview

The purpose of this unit is to provide 5th grade students with additional opportunities to extend their knowledge of probability. Through playing, analyzing, and creating games of chance using spinners and dice, students will explore the complexities of probability theory. This unit is intended as a supplement/support for the concepts set out in the School District of Philadelphia's core curriculum. The length of the unit can be flexible. Activities can be implemented at the conclusion of a unit of study on probability or interspersed throughout, whenever they are considered applicable.

Rationale

Mathematicians define probability as a way to describe the different levels of possibility of an event. Some events are impossible, while others are sure to happen. Most events though, fall somewhere along a continuum between these two extremes. The rest of us, however, understand probability as a world of uncertainties – something we must deal with every day. What's the chance of rain today? Should I take my umbrella? What is the likelihood of making most of the forty lights on the drive into work today? We make decisions about our meals based on medical researchers' predictions on the higher chance of eating a certain way; we buy insurance to protect us from the chance of loss and deny the chance of loss by buying lottery tickets and playing games in casinos. Quantum mechanics, a fairly new form of physics is based on the idea that no event, particularly what happens inside an atom can be guaranteed. Physicists can only predict the probability of certain events happening within that atom. The existence of this science demonstrates that statistics and probability are at the heart of our understanding of the universe (Vortman).

Probability theory has a long history dating back as far as the Renaissance. It arose, not surprisingly, as a result of peoples' interest in gambling and games of chance. Girolano Cardano (1501-76) was a medical doctor who was overly fond of gambling. Luckily, he was also a mathematician who was able to use his knowledge to help further his riskier pursuits. In Liber de Lundo Aleae (Book on Games of Chance) he set out two major ideas in probability theory that endure to this day. He counted the number of favorable cases and compared that with the total possible outcomes. (This is how we calculate theoretical probability.) He also proposed assigning a number from 0 to 1 to the probability /likelihood of an outcome. A 0 is assigned to an event that will never occur, while a 1 is assigned to an event that will definitely occur. Fractions are assigned to the possibility of the certain outcomes in between. Swiss mathematician Jakob Benoulli proved, in the late 17^{th} century the Law of Large Numbers: if we do an experiment often enough, the probability of that event will be an accurate measure of the theoretical probability. One hundred years later, again influenced by an interest in gambling, the mathematical theory of probability was further advanced by the work done by Pascal and Fermat. Pierre-Simon Laplace in the late 1700s defined the probability of an event to be the fraction:

P (Event) = F/T

(F being the favorable outcome and T being the total possible outcomes)

Finally, in 1933, a monograph by Russian mathematician A. Kolmogorov set out the approach that forms the basis for the current theory and application of probability.

Given that the probability of an event is a measure of the likelihood that an event will occur, how then, is that measured? There are two ways to determine the probability of an event: through logical analysis of the event, which is known as theoretical probability and through collecting data through experiments, which is known as experimental probability. Experimental probability and theoretical probability are related. The experimental probability of an event is based on experimentation or simulation and uses relative frequency to determine the likelihood of an event. The theoretical probability of an event is based on the analysis of the sample space (listing the outcomes of a single stage experiment) and uses symmetry, number or simple geometric measures to determine the likelihood of an event. The relationship between the two concepts arises from the fact that for a given event, experimental probability will more closely approximate theoretical probability as the number of trials increases. (Jones, et al 148). For example:

If you have a fair coin – one side heads, the other side tails, there are two possible outcomes when you toss it: one head, one tail. Each of the outcomes is equally likely, so each has a probability of ½. In theory then, out of 20 tosses, heads should appear ten times and tails, ten times. This is theoretical probability and is expressed:

Number of Outcomes in the Event/Number of Possible Outcomes

Now, what happens when you actually do toss the coin 20 times and record the data? It is most likely that you will not get heads ten out of twenty times. It could be 4 out of 20, (4/20) 6 out of 20, (6/20) 14 out of 20 (14/20). These are very different outcomes from those expressed in theoretical probability. These ratios are called relative frequencies and are expressed:

Number of Observed Occurrences of the Event/Total Number of Trials

If you continue to toss the coin many more times and record the outcomes, the relative frequency begins to come very close to the theoretical probability of the coin landing on heads (The Law of Large Numbers in action) and we can be pretty confident of the results. This interconnectedness also highlights another basic, but powerful tenant of probability theory: Probability is not about finding out what will happen on a particular trial or event. It is more about what will happen over many trials and using that to make predictions over the long run.

Experimental probability, theoretical probability, and the relationship between them are very important foundational concepts for students to grasp, but they are not always immediately apparent to elementary and middle school students. For this reason, students need many and varied activity- based experiences in order to develop a firm understanding of how probability works.

A basic grasp of probability and statistics, or risk literacy, as it is sometimes referred to, is critical to understanding and making choices about such everyday, but complicated matters such as science, politics, insurance, health, and money. As the Internet transformed access to information, it is more important than ever to teach people how to best interpret data. Understanding probability concepts early on may help children weigh life's odds and make sounder decisions as they make their way through the overwhelming maze of information (Spiegelhalter).

Probability not only presents real life mathematics - possibly more so than most other branches of mathematics, it also connects to many areas of mathematics along the curriculum continuum. Probability and statistics are studied in the upper grades, through Calculus. In the elementary grades, however, the connections are very strong to fractions and percents. The probability of an event is given as a number from 0 to 1; spinners have fractional parts; percents make comparing ratios that don't have the same denominator a lot easier. Comparing ratios requires students make use of their proportional reasoning skills as well. Data analysis is another area that connects handily to probability. In the course of conducting experiments to determine probability, students are collecting data and analyzing it to determine is predictive value. Additionally, work with probability enhances students' problem solving skills, especially as they work on experimental and theoretical tasks and allows them to apply their computational skills in a context. Finally, it can provide variety and challenges to students as they pursue their study of mathematics (Van de Walle).

If students are to understand probability at a deeper level in high school and college, then the concepts and skills necessary for this type of mastery need to be taught in the earlier grades. Brunner noted, "If the understanding of number measures and probability is judged crucial in the pursuit of science, then instruction in these subjects should begin as intellectually honestly and as early as possible in a manner consistent with the child's form of thought. Let topics be developed and redeveloped in later grades. (Brunner 53-54). In fact, the NCTM, in its 2000 edition of Principles and Standards for School

Mathematics, recommended that the teaching and learning of probability should begin as early as kindergarten.

Because the ideas and methods of probability are so prevalent in all aspects of our lives, this strand of mathematics has had, in recent years, an increasingly greater presence and importance in mathematics curricula in the elementary grades.

Probability is not an easy topic to teach. It is, as Godino notes, "a notoriously difficult topic to teach even for high school and college students in part because students' intuitions are not always in sync with the mathematics of probability." The following snippet from a group of third graders engaged in a probability task gives us an insight into the hurdles teachers at all levels face.

Students in a third grade class made gumball shakers and put 4 blue and 2 red gumballs in each. Their task was to determine which gumball, the red or the blue one was most likely to come out of the shaker. One group's discussion follows:

> Student 1: I predict it'll be red
> Student 2: No, it's going to be blue because there are more blue.
> Student 3: Yeah, it'll be blue because the blue are at the bottom.
> Student 1: Red is my favorite color. It always comes up for me.
>
> (Jones, et al p.147)

Through this conversation, we catch a glimpse of the varying levels of probabilistic reasoning that can exist in a classroom. Counterintuitive results in probability are found at very elementary levels, whereas in other branches of mathematics counterintuitive results are encountered only when working at a high decree of abstraction. Borovcnik and Peard cited in Godino, et al) Reasoning about probability is different from logical or causal reasoning. In logical reasoning a proposition is always true or false. A proposition about a random event is true or false only after the experiment is carried out. . Understanding of probability is not simply composed of solution strategies and procedural knowledge, it also requires a probabilistic thinking process along side other mathematical skills. Students have developed intuitions about chance events from their earliest experiences with board games, but these ideas are not always compatible with the mathematics. Because students' probabilistic reasoning can be diverse, idiosyncratic, and even subject to a certain mystique, the task of teaching probability can be particularly challenging. (Jones)

A review of the literature on teaching probability concepts consistently supports the notion that students should not be passive learners. (No revelations here.) Rather than copy down rules and definitions, students should be provided with many opportunities to explore through activities and simulations. These informal concrete experiences help students establish a foundation upon which more formal abstract ideas can later be built. Spinners, dice, coins, colored balls and blocks are manipulatives that have been staples of the mathematics classroom for a long time. Now though, we have another tool at our disposal – virtual manipulatives. These are available to classrooms through nlvm.usu.edu

– a site created as a library of virtual manipulatives developed at Utah State University. The site allows students to spin spinners that can be designed to represent any situation. Though there are no dice, spinners can be reconfigured to represent number cubes. There are many advantages to using these manipulataives in the course of teaching probability. Students think it's really cool (Does more need to be said?), so attention and motivation are enhanced. Because there is less focus on the physical manipulation of the spinner and the vagaries associated with a manual spin, there is more time to build sample space, so students can see that a small number of trials can contain wide variations, but through many trials, better estimates about what will happen in the long run can be made, thereby enabling students to explore the law of large numbers to greater effect. There is increased opportunity for students to make predictions of theoretical probability and compare these with the experimental probability results and in so doing, be able to correct their misconceptions. Van de Walle, however, offers a caveat to this. He feels that using technology exclusively may mask what is happening, such as how sample spaces are generated. This can, in turn, interfere with students' understanding of probability. Having students actually spin, toss, and roll concrete materials is a recommended first approach.

Students' intuitions about probability are powerful constructs and can adversely affect teaching and learning regardless of the clarity and logic of instruction. Therefore, throughout any unit on probability, students need lots of hands on experiences in order to challenge and correct these misconceptions.

Objectives

Probability is a complex topic with many concepts that are interwoven/interrelated and seemingly counterintuitive. The focus of this unit is to build on several key concepts students should have a firm grasp of in order to do the more difficult work ahead of them in late middle school and high school.

These are:

-sample space (listing outcomes of a single stage experiment)
-experimental probability
-theoretical probability, and the relationship between them
-develop an understanding of the possible outcomes in a situation
-understand what fairness means in the context of probability
-determine whether games of chance are fair or unfair and how to make unfair games fair
-calculate the odds and probability as a long-term phenomenon.

Additionally, students' prior conceptions will be challenged and corrected. They will come to understand, among other things, that chance, as van de Walle notes, has no memory. Just because heads came up five times in a row, the probability of getting heads on the next toss is not at all affected – it still remains 50/50.

Strategies

Since students will be meeting the objectives through playing, analyzing, and creating games, the primary strategy used throughout this unit is hands-on experimentation. Working collaboratively in pairs or small groups, students will use a variety of manipulatives – concrete or virtual to simulate the gaming situations they will be exploring. Students will keep individual notebooks in which they will record their work, write down questions they have, and respond to questions centering on the work they will be doing.

Classroom Activities

Students will be introduced to the unit and its theme- using games of chance to help student council raise money for the school. They will understand that this will involve playing games, assessing the fairness of games, changing unfair games to fair ones, determining the odds for particular events and how to determine the payout for particular wagers. They will also calculate the extremes - what is the most money the students could earn, what is the least and how they can use their knowledge of probability to help student council earn the maximum.

As a culminating activity, students will create their own game, calculate the probabilities of winning, and then play the game with several classmates. Students will then discuss with the designer what they liked about the game, and how, if necessary, it could be improved. Students will then make revisions to their games

<u>Part 1 – Explore Fairness in Games</u>

Begin by asking students to respond in their notebooks to the following questions:

> What do you think it means for a game to be fair? What conditions must be present?
>
> Would you want to play an unfair game, even if it was in your favor?

Have students share their ideas and record them on chart paper. This will be saved for the end of the unit and discussed later.

<u>*Candyland* Activity</u>

As suggested by D. DeTurck and adapted from "Connected Mathematics Unit – What Do You Expect?" Use to review basic concepts of probability and to get to the idea of fairness in probability.

Materials:
Candyland game
In lieu of the game, use colored cubes or tiles
Non transparent container or bag

Student notebooks

Poll the class to see who has played the game *Candyland?* (Most likely all have.)
Ask if they think this is a fair game and why.

Take out all the picture cards and place the remaining color cards in a container.
Tell students there are red, purple, yellow, blue, orange and green cards in the container and that they will have to predict the fraction of each color in the container, without emptying it.

One by one have a student pick a card, record the color on the board then return the card to the container.
Once all students have picked ask:

How many cards of each color did the class pick?
Which color are there most of? Which the least?
Predict the fraction of each color.

Finally, look at all the cards. Find the actual fraction of cards for each color. How close so these come to your predictions?

Discussion points - Students answer in journals and discuss as a class.

Is each card equally likely to be picked?
Is each color equally likely to be picked?
What is the probability of picking a white card?
How many cards would need to be added to make the probability of
Picking blue, for example, ½?
Is this game fair? Why/Why not

Discuss/explain how fairness in probability is quite different from the everyday meaning of fairness, which we understand as something that is just or honest. In probability, fairness means there is nothing in the rules that would favor one player over another – everyone has an equal chance of wining. Conversely, it does not mean that each person will win the same number of times, nor does it mean that the player who wins this time will win the next time.

<u>Spinner Games</u>

Sometimes in complex situations it can be difficult to determine if a game is fair or not. But students of this age can develop an understanding of fairness by gathering data on winners and losers to determine if each player had an equal chance. (Bright, et al 50) Collecting data is simple- play the game and record the results. The important thing here is to play many games.

<u>I'll Race You</u> (Adapted from Investigations)

Materials:
One equally divide spinner per pair of students

Recording sheet and pencil

In this game students use an equally divided two- color spinner. (Green/White)
Players decide who goes first by tossing a coin. Winner chooses his/her color.
Players take turns spinning the spinner
Green player gets a point if the spinner lands on a green section
White player gets a point when the spinner lands on a white section.
You get a point whenever the spinner lands on your color, no matter who spins the spinner.
On a score sheet with ten slots, record an x each time your color comes up
First player to fill in an x on every slot wins that round
Play ten rounds

Sample score sheet: Round # 1

Green / White

__x_____/_______

After reading the rules, but before students play, ask:

Is this a fair game?
Which player would you rather be?
What's the probability of the green player winning a round?
What is the probability of the white player winning a round?
After ten rounds, how many rounds will each player have won?

Create 2 line plots on the board - one for each color. As students complete a round, have them record the win on the line plot.

When the whole class has completed all rounds and recorded their results, look at the line plots and discuss the data. Ask students to consider:

How can we represent the results?
Were they surprised by the results?
Did one color win more often than another? Does that mean the game is unfair?
What kind of results would we get if we played many more games?

At this point, have student pairs create a replica of the manual spinner using the virtual manipulatives site and play 100 rounds and record the results.
Use the results from these activities to reinforce the concepts of experimental and theoretical probability and the Law of Large Numbers.

How are these results different from the class's ten rounds?
What percent of the time does each player win?
Use these results to reinforce concepts of experimental and theoretical probability

Spinner Two

Using the virtual manipulative library, have students create 2 spinners.

Each will have 12 sections with 3 different colors
5 green, 5 blue, 2 red

Spinner 1 should have the sections arranged as follows:

R<B<G<B<G<G<R<B<B<G<B<G<

Spinner 2 should have the sections arranged as follows:

G>G>G>G>G>B>B>B>B>B>R>R

Ask students to analyze these spinners.

Would you have a better chance of landing on green in spinner 2?
Would you have a better chance of landing on blue in spinner 2?
What is the probability of landing on green in each spinner? Of landing on blue?
Are the spinners the same from the probability standpoint?
Could these spinners be used in a fair game?
If you wanted to redesign the spinner and you could not take any sections away but you could add sections, how many red sections could you add?

<u>Number Cube Games</u>

Materials:
Two different colored dice or 6-sided 1-6 number cubes
Paper and pencil for each student pair

<u>Addition Game</u> (2 players)

Both players take turns rolling the number cubes
Player 1 gets a point of the sum of the faces is odd
Player 2 gets a point if the sum of the faces is even
Play continues for 36 rolls
Players record the results of each roll

Introduce the game with a few demonstration rolls and sample result recording.

Ask students if they think this is a fair game and why.
If students don't think so, ask them which player it favors.

Have pairs play the game, keeping track of their results
Afterwards have students record the answers to these questions in their notebooks:

Based on your data, what is the experimental probability of rolling an odd sum? An even sum?
List all the possible pairs of numbers you can roll with two number cubes.
What is the theoretical probability of rolling an odd sum? An even sum?
Is this a fair game?

Create a line plot on the board using the possible outcomes of adding the two cubes.
Ask a pair for their results and record it on the plot.
Analyze the probabilities of rolling an even sum and an odd sum.
Discuss which sums were more likely.

Combine the class data in a similar way and discuss the experimental probability of rolling an odd sum and an even sum and how they found these probabilities.
Based on the data, is the game fair?

Students may have had difficulty listing all the possible pairs of numbers that can be rolled with two number cubes. At this point it would be very useful to introduce them to the following chart as a means of finding all the possible sums (the sample space).
Have students copy the chart into their notebooks as an example and for future reference.

Number Cube 1

	+	**1**	**2**	**3**	**4**	**5**	**6**
	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
Number Cube 2	**3**	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

This will help the class compare the theoretical probabilities to their experimental probabilities. To help them analyze this and to review other concepts ask:

How many ways are there to get an even sum?
How many ways to get an odd sum?
Does each player have an equal chance of winning this game?
What is the probability of getting a sum that is a prime number?
A multiple of 5?
A multiple of 2 and 3?
A factor of 24?
A multiple of 15?
Doubles?
A 7?
An 11?

Keep this chart for use later when students are determining what dollar amounts to assign each roll in the student council game.

Have students recreate this simulation using virtual manipulatives and 1000 throws of the cubes. Record the results.

How are the results the same as those of the class?
How are they different?

Multiplication Game (Not so Fair)

This is similar to the addition game, but points are scored based on the product of the rolls.

Player 1 gets a point if the product of the roll is odd
Player 2 gets a point if the product of the roll is even.
Play continues for 36 rolls and students record the results of each roll
Player with the most points wins.

Have students play the game and when done, answer the following questions in their notebooks:

Based on your experimental data, what is the probability of rolling an odd product? An even product?
What is the theoretical probability of rolling an odd product? An even product?
Do you think this game is fair? Why or why not?

Make a line plot with the possible products from rolling two number cubes and record the class results for each time a product came up.

Compute the class's experimental probability for the rolls.
Have the class create a chart for the product possibilities similar to the one created for the addition possibilities.

Using this, discuss the theoretical probabilities of rolling an odd product and of rolling an even product.

Discuss the differences between the experimental and theoretical probabilities.
As with the addition chart, use it to extend and review other concepts:

What is the probability of getting:
A prime number?
A square number?
A multiple of 2 or 3?
A factor of both 12 and 15?

Ask students if this is a fair game? If they decide it isn't have them redesign the game to make it fair.
Once students have had a chance to make revisions, ask for a pair of volunteers to share their new and improved game. Have the class play several rounds to get a feel for the game and ask them if they think everyone now has an equal chance of winning and why they think so.

<u>Part 2- Making Money for Student Council</u>

For this section, students will use the charts created for the number cube addition and multiplication games. They will place the bets and play the games, then determine the money (the least and the most) that student council could make.

<u>Game 1</u>

Students have already calculated the probabilities for rolling the sums for each of the following events and the pay-outs are listed accordingly:

Doubles	1/6	5 times the bet
Primes	15/36 (5/12)	2 times the bet
Odds	½	1 times the bet
Evens	½	1 times the bet
7	1/6	4 times the bet
11	1/18	10 times the bet

These outcomes will be printed on a mat and a player will choose an outcome and place a token on that space.
Each player will pay $2.00 to play. The player will then throw the number cubes.
If the expected outcome comes up, the player will receive the posted pay out.
If it doesn't, the money remains in the bank.

If 72 people place a bet on each outcome, how will student council fare?
Will they make money, lose money, or break even? Have students compute the "house" take.

<u>Game 2</u>

Student Council can use the addition game in a simpler form.
Each player will pay $2.00 to play. The player chooses an odd or an even number for the outcome of the throw. If the player wins, the payout is $3.00.

If 100 people play, will student council make money, lose money, or break even?
What is the most they could win?
What is the least they could win?
Students can use the virtual manipulative site to help with these decisions.

Game 3

Uses the data from the number cube multiplication game.

Each player will pay $2.00 to play.
The player chooses to play for an odd product or an even product.
If the player chooses odd he receives 3 times his bet
If he chooses even, he gets double his bet

If 100 people play, will student council make money, lose money or break even?
What is the most they could win?
What is the least they could win?

Part 3 – Make Your Own Game

As a culminating activity, students will design their own game of chance.

Before they begin their work on this project, however, bring out the chart paper from the beginning of the unit and review with them their original responses to the question of a game's fairness. Discuss what they thought originally and what they think and know now. These insights should guide them as they design their games.

Requirements:

The games can use dice, spinners, colored blocks, or coins.

The game must be fair and students will need to explain how they know what they've created is fair.

Students will calculate the experimental and theoretical probabilities associated with the game

Students will make a board, mat or whatever playing surface is needed
Consideration should be given to the size of the playing field, interesting and colorful graphics and clarity of the playing field.

Student designers will write out the rules for the game in clear, concise language.

Students will then have an opportunity to "test drive" their games by playing with other students in order to collect data about their games.

As the games are played, students will make decisions about their own games as well as their classmates' games.

They will consider features such as: is it fun to play, is it easy to play without being boring or overly repetitive, is it fair, would you play it again if you didn't have to.

Students can offer suggestions to their classmates and game designers can then make modifications before submitting the final version for a grade.

Bibliography

Ashline, George, and Marny Frantz. "Proportional Reasoning and Probability: A Professional Development Context for Mathematics Educators." *Connect Magazine*. Nov/Dec.2009: 8-10. Print.

Authors investigate how using proportional reasoning in a variety of contexts helps students better understand probability and statistics concepts.

Aspinwall, Leslie, and Kenneth Shaw. "Enriching Students' Mathematical Intuitions With Probability Games and Tree Diagrams." *Mathematics Teaching in the Middle School*. December 2000: 214-220. Print.

Authors describe how using probability games helped students foster their intuitions connected with the concept of "fairness" in the outcomes of compound events.

Beck, Shari, and Vanessa Huse. "A Virtual Spin on the Teaching of Probability." *Teaching Children Mathematics*. May 2007: 206-211. Print.

This article examines the many advantages the authors discovered in using virtual manipulatives rather than the more conventional concrete manipulatives in teacing probability.

Bretzlauf, John. "Essay on Data and Chance." *Everyday Mathematics Teacher's Reference Manual, University of Chicago School Mathematics Project*. Ed. Max Bell. Chicago: SRA, A Div. of The McGraw-Hill Companies, 2002. Print.

This introductory essay offers a basic explanation of the concepts of data and probability.

Bright, George, John G. Harvey, and Margariete Wheeler, "Fair Games, Unfair Games." *Teaching Statistics and Probability, 1981 Yearbook*. Shulte, Albert, P. Reston: National Council of Teachers of Mathematics, 1981. Print.

This article examines mathematical fairness and unfairness and shows how students' notions of these concepts can be refined through playing appealing classroom games.

Burns, Marilyn. *About Teaching Mathematics, A K-8 Resource*. 2nd. Sausalito: Math Solutions Publications, 2000. 59-77. Print.

An excellent resource for teachers no matter their background in math. It provides classroom activities to help teachers to teach math through problem solving. It also provides insights into thinking mathematically and how children best learn math.

Chick, Helen, and Monica Baker. "Teaching Elementary Probability: Not Leaving It To Chance." *Australian Primary Mathematics Classroom*. March 2007: 220-228. Print.

The authors recommend that teachers introduce probability concepts through activities and simulations not through abstractions.

Curcio, Frances, R.. "Developing Concepts in Probability: Designing and Analyzing Games." *Mathematics in the Middle*. Ed.. Larry Leutzinger. Reston: National Council of Teachers of Mathematics, 1998. Print.

Teachers in a middle school classroom designed tasks for their students that would engage them in the study of probability and allow the teachers to assess student progress and further instructional decision making.

Godino, Juan, and Carmen Batanero. "Training Teachers to Teach Probability." *Journal of Statistics Education*. 12.1 (2004): 325-335. Print.

Authors analyze why the teaching of probability is difficult and make suggestions for needed content instruction for teachers.

Gurbuz, Ramazan. "The Effect of Activity-based Instruction on Conceptual Development of Seventh Grade Students in Probability." *International Journal of Mathematical Education in Science and Technology*. 41.6 (2010): 743-767. Print.

Through his work with seventh grade mathematics students, the author observed that students developed stronger conceptual understanding of probability concepts when taught through hands on instruction, rather than lecture and note taking..

Jardine, Dick. "Looking at Probability Through A Historical Lens." *Mathematics Teaching in the Middle School*. Sep.2000: 50. Print.

Thins is an interesting article taking a historical look at probability and games of chance and some of the mathematicians involved in the development of probability.

Jones, Graham A.. "Understanding Students' Probabilistic Reasoning." *Developing Mathematical Reasoning in Grades K-12*. Ed.. Lee V. Stiff. Reston: National Council of Teachers of Mathematics, 1999. Print.

There is a range of probabilistic thinking exhibited by students in any classroom. This article explores the pattern of growth in probabilistic reasoning that students go through and provides a framework that teachers can use to construct appropriate probability tasks to access and monitor students' reasoning.

Lappan, Glenda. "What Do You Expect: Probability and Expected Value". *Connected Mathematics*. Comp. James T. Fey. Glenview: Prentice Hall, 2002. Print.

This is the probability unit in the Connected Mathematics curriculum.

Shulte, Albert, and James Smart. *Teaching Statistics and Probability*. 1st. Reston, Virginia: The National Council of Teachers of Mathematics, 1981. Print.

NCTM's yearbook dedicated to exploring the topics of statistics and probability. It is a compilation of articles on the topic from making the case for teaching Statistics and probability to classroom applications K-12.

Sorto, Alejandra, and Alexander White. "The Gumball Machine: Linking Research and Practice About the Concept of Variability." *Journal of Statistics Education*. 2008: 267-275. Print.

The authors modified an existing NCTM lesson. It gives students an opportunity to explore and discuss the variation which occurs in sampling.

Wittenberg, Lucy. "How Long Can You Stand On One Foot?." *Investigations in Number, Data and Space*. Ed.. Susan Jo Russell. Glenview: Pearson Scott Foresman, 2008. Print.

This is the grade 5 unit in the Dsata Analysis and Probability strand of this curriculum. It further develops ideas about collecting, representing, describing and predicting the likelihood of events.

Taylor, Felicia. "Why Teach Probability in the Elementary Classroom." *Louisiana Association of Teachers of Mathematics*. 2001: 26-37. Print.

Because probability presents real life mathematics and also connects to many mathematical areas, the author makes the case for teaching probability as early as possible and in a manner "consistent with the child's form of thought.".

Tomlinson, Stephen, and Robert Quinn. "Understanding Conditional Probability." *Teaching Statistics and Probability*. 19.1 (1997): 2-8. Print.

This article offers a new approach to teaching the difficult concept of conditional probability.

Van De Walle, John. *Elementary and Middle School Mathematics: Teaching Developmentally*. 6th ed. New York: Pearson Education, Inc, 2007. 475-491. Print.

This is an excellent resource for teachers. It elucidates mathematical concepts, and combines that with an understanding of how children really learn math and how to further that process through problem based work in the classroom.

Watson, Jane, and Jonathan Moritz. "Fairness of Dice: A Longitudinal Study of Students' Beliefs and Strategies for Making Judgements." *Journal for Research in Mathematics Education*. 34.4 (2003): 270-304. Print. *Developmentally*. 6th ed. New York: Pearson Education, Inc, 2007. 475-491. Print.

The authors examined four levels of beliefs about fairness and four levels of strategies for determining fairness. They revisited these concepts with the same students four years later.

Zawojewski, Judith. *Curriculum and Evaluation Standards for School Mathematics Addenda Series, Grades 5-8: Dealing With Data and Chance*. 5th. Reston: The National Council of Teachers of Mathematics, 1991. Print.

This is one of a series of books that provides teachers with materials to support the implementation of NCTM standards in their classrooms.

Bibliography for Students

Cushman, Jean. *Do You Wanna Bet? Your Chance to Find Out About Probability*, New York: Clarion Books, 2007. Originally published in 1991.

Two boys become involved in everyday situations involving probability. Through this narrative, students might create probability simulations or design probability experiments to test the outcomes in the text.

Grossman, Bill. *My Little Sister Ate One Hare*. New York: Crown Publishing, Inc.,1998.

This will appeal primarily to younger children, but upper elementary kids will enjoy it for its gross factor. This is essentially a counting book in which a little girls eats a variety of disgusting creatures and then throws them all up. It has been used to create an introductory lesson on probability. If one of the things the sister threw up is picked up, what is the probability that it is a …. Students can explore experimental and theoretical probability concepts.

Resources

Virtual Manipulative sites
www.Shodor.org/interactivate/activities/ExpProbility
This website provides virtual manipulatives as well as tutorials relating to all areas of mathematics

http://www.pbs.org/teachersource/index.htm
On this website, there is a large selection of problems on real life applications of probability and statistics concepts aw well as lesson plans.

http://nlvm.usu.edu/en/nav/frames_asid-305_g_3_t_5.html
This is an extensive resource of more than 5wenty-five virtual manipulatives developed at Utah State University.

In addition to/or in place of these virtual manipulative sites classroom resources should include:

Number cubes/dice
Spinners – clear plastic so that sectors of varying size can be created as needed
Centimeter cubes/or linking blocks in a variety of colors
Candyland game (Optional)
Notebooks

Standards Addressed

2.4 – Mathematical Reasoning and Connections

B. Use models, number facts, properties and relationships to check and verify predictions and explain reasoning.
C. Draw inductive and deductive conclusions within mathematical contexts.

2.5 – Mathematical Problem Solving and Communication

A. Develop a plan to analyze a problem, identify the information needed to solve the problem, carry out the plan, check whether an answer makes sense and explain how the problem was solved.
E. Select, use and justify the methods, materials and strategies used to solve problems.

2.6 – Statistics and Data Analysis

B. Describe data sets using mean, median, mode and range
C. Predict the likely number of times a condition will occur based on analyzed data.
E. Construct and defend simple conclusions based on data.

2.7 – Probability and Predictions

A. Perform simulations with concrete devices (e.g., dice, spinners) to predict the chance of an event occurring.
B. Determine the fairness of the design of a spinner.
C. Express probabilities as fractions and decimals
D. Compare predictions based on theoretical probability and experimental results
E. Calculate the probability of a simple event
F. Determine patterns generated as a result of an experiment.
G. Determine the probability of an event involving “and”, “or,” or “not”.
H. Predict and determine why some outcomes are certain, more likely, less likely, equally likely or impossible.
I. Find all possible combinations and arrangements involving a limited number of variables.