Building Numeracy and Dismantling Math-Phobia Through Problem-Solving

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Rationale

I'm particularly interested in the interplay of the philosophical methods - including challenging assumptions, analyzing arguments, making connections, questioning intuitions – and what people might normally think of when they imagine the thinking that happens in a math classroom. But possibly contrary to popular belief, math is the ability to solve problems requiring independence, judgement, originality, and creativity. (Polya, 1962) Unless presented frequently with concrete applications of more abstract mathematical questions, I find that students focus on repeating rote procedure correctly and finding the answer that they think I want to hear. Students focus so much on performing the "right" process to get the "right" answer, that they often neglect to think about what the answer means in the context of a problem or consider whether the numerical answer(s) at which they've arrived really make sense in that same context with respect to the queries or requirements of the original prompt. Sometimes, students develop an almost automatic response to certain "clues" in problems or prompts, and they fail to recognize what is really being asked of them as a response. Even the most creative of my students often exhibit thinking that indicates a dependence on standard algorithms without consideration for possible alternative strategies or solutions.

Additionally, without practice and educator-assisted development of various critical thinking strategies, students are rarely able to transition from a concrete application and understanding to mathematical abstractions with which they have little or no concrete familiarity, and maybe surprisingly, vice versa as well. For example, if I teach students explicitly, the step-by-step procedure for solving problems with three variables, some of them will inevitably "get it" by memorizing the process and practicing it repetitively. Those same students might struggle with a more concrete application in a familiar context, whereas many of the students who struggled to master the procedure might have a much easier time with the more concrete application (though without using the same procedure). I also see students struggle to draw connections between different topics throughout the duration of a math course and this inability to see relatedness among various topics hinders their ability to really grasp mathematical content in a way in which they can effectively reason through an unfamiliar question or prompt that would use skills with which they might be familiar. I want to my students' thought processes to involve a recognition of mathematical structure in a variety of familiar and unfamiliar problems, a confidence that their reasoning is sound with relation to a problem, and also the ability to question their own reasoning and the reasoning of others if the answers or justifications by which they or others have reached an answer seem questionable or unsound. These skills will both help students work their way through problems effectively and creatively, and also reflect on the mathematical soundness of their own work and the work of others. If students are better able to recognize when answers don't make sense, and they have the confidence to explain why an answer doesn't make sense, they will have a far easier time of correctly identifying whatever mistake or mistakes were made in the process of arriving at an incorrect answer, and will be able to more effectively rectify those mistakes.

I believe that part of the reason that students fail to find their own errors and seem to value procedure over logic has to do with habit and experience, but is also due to a lack of confidence. As with most things, confidence comes from experience, and is most effectively built in an environment where students feel safe to make mistakes and take risks. If students learn to use the above philosophical methods and apply them to mathematics, I can gradually give them greater responsibility and more autonomy to explore math by use of critical analyses and questions. With greater confidence in their ability to effectively utilize these skills, my hope is that students will rely less on seemingly arbitrary mathematical procedures and algorithms and will develop the confidence and ability to tackle significantly more complex topics with a deeper understanding of the work that they are doing. These skills will be particularly useful to them in my class, but will be of additional value to them in all of their math classes at the high school level and beyond.

Audience

This unit is designed to be implemented piecemeal throughout the course of an Algebra 1 course for freshmen students. The course meets five days per week for approximately 50 minutes each day, but the implementation of the pieces of this unit is intended to take place on a weekly or twice weekly basis. The unit is designed for students at the Academy at Palumbo, which is an academic magnet school in the School District of Philadelphia. Because the magnet school falls into the "special admit" category in the school district, students apply and travel from all over the city, which results in a very diverse student body with regard to ethnic, cultural, and academic backgrounds. Accordingly, the tasks in the unit are designed to adhere to the "low floor, high ceiling" framework**.** "[Low floor high ceiling tasks]…are tasks that are easily accessed by students with varying backgrounds, that can be seen and solved in different ways and that can be taken to high levels." (Boaler, 2016) Use of tasks that have a low floor and high ceiling will allow me to both engage students who have historically struggled in math classrooms and challenge the most mathematically advanced and interested students.

A Note on Standardized Testing

As of May 2018, students who will graduate in 2020 or after will be required to pass the Keystone Algebra 1 exam in order to graduate. Though this graduation requirement has been postponed every year since its inception, I attempt to take into instructional consideration the possibility that students may need a passing score in order to graduate.

However, “teaching to the test” can take up weeks or even months of instructional time over the course of a school year, in addition to the time required for the administration of the actual test(s). This curriculum is designed to equip students with skills that allow them to generate mathematical knowledge independent of prescribed procedures and rules; these same skills will be of value to students in a standardized testing context, but are not included solely for the sake of test proficiency.

Background

Math is a tool that allows students to better understand themselves, their community, and the place of both of the former in a worldwide context. Understanding math is empowering, and is a necessity for the informed and critically-thinking citizen.

Enduring Ideas About Education

John Dewey, the father of progressive education, believed that “…all which the school can or need do for pupils is to develop their ability to think.” (Dewey, 1916) The ability to think critically and independently, rather than rote memorization of information or procedure was, for Dewey, the sole reason for schooling. In his 1916 *Democracy & Education*, Dewey said,

> …skill obtained apart from thinking is not connected with any sense of the purposes for which it is to be used. It consequently leaves a man at the mercy of his routine habits and of the authoritative control of others, who know what they are about and who are not especially scrupulous as to their means of achievement. And information severed from thoughtful action is dead, a mind-crushing load. Since it simulates knowledge and thereby develops the poison of conceit, it is a most powerful obstacle to further growth in the grace of intelligence. (p.158-159)

In his *Pedagogy of the Oppressed*, Paulo Friere argued that much of education relied on a “banking” conception of education, where students were merely receptacles for information determined and delivered by the teacher.

> Education becomes an act of depositing, in which the students are the depositories and the teacher is the depositor. Instead of communicating, the teacher issues communiqués and makes deposits which the students patiently receive, memorize, and repeat (p. 58) Translated into practice, this concept is well suited to the purposes of the oppressors, whose tranquility rests on how well men fit the world the oppressors have created, and how little they question it. (pp. 62-63)

Teachers’ use of “banking” methods of education leaves students with only the capability to regurgitate information, and deprives them of the skills they need to be contributing and critical citizens. He believes that this failure is intentional in order to maintain a populace incapable of demanding rights and questioning power. In order to empower students and facilitate the “emergence of consciousness and critical intervention in reality,” (Freire, 1968) Friere suggested a problem-posing approach to education in which students construct knowledge in relation to themselves and their worlds, rather than simply acting as vessels to be filled with prescribed topics and facts.

The Organization for Economic Co-Operation and Development administers a test entitled “The Programme for International Student Assessment,” or PISA, for short, in order to compare the academic achievement and growth of 15-year-old students in participating countries. The data available for this assessment - a century after Dewey’s *Democracy & Education* and nearly 50 years after Friere’s *Pedagogy of the Oppressed* - implies that students are more likely to experience success in solving difficult problems if they experience more student-driven instruction rather than constant teacher-centered learning. The data also shows that students in the highest performing countries rely much less on memorization strategies than do students in lower performing countries, according to self-reporting by those students. (PISA, 2016) In order to ensure that students have a deep and lasting understanding of the mathematics that they’re learning, they must be given the opportunity to make sense of the ideas, particularly in contexts that are meaningful and familiar. Additionally, if schools and educators want to best facilitate learning, students must be encouraged to be creative in their attempts to understand and apply concepts, and to think about the math itself rather than what a teacher told them to think about the math. When we treat students as individuals capable of independent and critical thought, it becomes evident that they are capable of far more than merely serving as receptacles for the information that teachers choose to deliver, and possess more potential as mathematicians and scholars than is usually assumed.

Standards vs Content: Meeting Instructional Goals and Shaping the Mathematical Self

The new Common Core State Standards for Mathematics (CCSSM) were designed to guide educators to deliver instruction that would allow for deeper understanding and student-driven instruction, particularly with regard to the teaching of mathematical thinking. The standards are intended not to teach students how to better memorize disconnected mathematical concepts and facts, but how to better understand, think creatively about, and apply them. ((National Governors Association Center for Best Practices, Council of Chief State School Officers , 2010)). Though there is ongoing debate in regard to all aspects of the standards (Russell, 2012), teachers often do not have much flexibility with regard to the overarching mathematical concepts that are required grade by grade. However, the CCSSM does not tell us how to teach or the content we must use to help students master the appropriate content standards. We can think about the standards as a set of errands: we have to go to the grocery store, drop off the dry-cleaning, and get a car wash – but it’s up to us to choose which to do first, what car wash to use, what groceries to buy at the store. Educators must chiefly know their students: students' strengths and learning needs, learning styles, interests and passions.

An individual teacher's beliefs and experiences about math instruction directly impacts their instructional choices, implementation of innovative practices, and the climate for mathematics learning in their classrooms. Additionally, a teacher's own beliefs about innovative practices and about education as either transmission or construction largely determines their instruction. (Stemhagen, 2011) Particularly in poor, urban school districts, there appears to be a resistance (among not only educators, but also among administration and "reformers") to a student-driven, constructivist approach to instruction and an undying adherence to the belief that students must be force-fed the traditional mathematical cannon in a repetitive and almost robotic fashion (see: Corrective Math, an entirely scripted curriculum program purchased by the School District of Philadelphia in 2010 that forced students from elementary through high school to participate in an arithmetic remediation program that involved call and response, clapping, and dozens and dozens of problems requiring absolutely no critical or creative thought. Students were placed into these classes based on their performance on a brief arithmetic test. This class took the place of an elective for all students who were required to participate.) Math education has evolved into something whose purpose is enabling more mathematical attainment rather than the development of students as competent, critically thinking, and participatory citizens. (Noddings, 1993) In a democratic mathematics classroom, students should be given greater agency in discovering, debating, and constructing their own mathematical understandings – teachers should question and support, but should refrain from delivering to students any conclusion that students could reach independently.

It is more socially acceptable to be innumerate than to be illiterate. Adults often brag or joke about being terrible at math, and this sends the message to students that the development of mathematical skill and understanding is not something of which everyone is capable. Gender and racial manifestation of stereotypes – boys are good at math, girls are good at literature, students of color typically struggle in math (as measured by standardized tests) – have been deeply embedded in the consciences of most, if not all, students from an early age. Additionally, curriculum companies write content independent of consideration for who their learners are in the world, which furthers the seeming divide in ability and aptitude. (Martin & Gholson, 2012) However, Spencer, Steele, and Quinn found that if stereotype threat were eliminated prior to evaluation, differences between men and women in performance on math tests was eliminated. (Spencer, Steele, & Quinn, 1999) With regard to racial differences in math achievement, school segregation and inequality is undeniably a substantial factor in differing achievement levels. But students' academic identities and commitment to STEM fields are antecedents to STEM participation and persistence. (Childs, 2017) Facilitating the development and support of diverse student mathematical identities will also require a shift in identity for most teachers – less an authority, more a facilitator; willing to make mistakes and question themselves, their pedagogy. If the teacher does not make use of a variety of strategies to meet the range of students' needs, constructivist practices can perpetuate bias and exacerbate problems of inequity rather than reduce them. (Allen, 2011)

Objectives and Strategies

Standards for Mathematical Practice

The vast majority of the content objectives are derived from the Common Core State Standards for Mathematics, but the Standards for Mathematical Practice will largely govern my instructional methods and choices:

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
9. Look for and express regularity in repeated reasoning

These standards are a "balanced combination of procedure and understanding" (National Governors Association Center for Best Practices, Council of Chief State School Officers , 2010) I intended this unit to be a year-long set of problems, that require students to use one or more of these standards in the context of a collaborative problem-solving effort. These problems may or may not have a single right answer, but students will not be given any examples or algorithms with which to solve the problem. Instead, they will be encouraged to discuss their ideas, work through (what I imagine will be) a series of incorrect answers/approaches, and eventually approach an understanding of the mathematical content with as little prescriptive, process-based instruction as possible with the aim of progressively increasing difficulty and student responsibility. I plan to spend less time on test prep and any instruction that remotely resembles "drill-and-kill" in order to dedicate more time in class (ideally 1x/week or 1x/biweekly) to allow students to struggle with, debate, and tackle these unfamiliar and usually challenging problems. There are other "mathematical practices" that are more specific and use more student-friendly language than those listed above – I will detail and refer to those throughout this unit – but the practices above are the framework around which I will plan my lessons, and accordingly, the problems I assign to students will guide or require them to use one or more of these practices.

Objectives: Using Habits of Mind

My students have not often or ever had the experience of a math classroom in which they are allowed or required to struggle. Because math has, for most students, been an experience of being either right or wrong, students either hesitate to take intellectual risks publicly or don't understand how to take that sort of risk. In order to best facilitate the development of a student who is willing and able to construct mathematical meaning for themselves and participate in collective or individual productive struggle, I plan to scaffold the development of specific "habits of mind" over the entire course of a school year. These habits of mind are in many ways an iteration of the Standards for Mathematical Practice, but written in more student friendly language and lend themselves more to appropriate objectives for lessons. They are intentionally broad, but the overall objective of this entire unit is that students learn, practice, and effectively use these habits. The habits are as follows:

1. Look for patterns: look for patterns amongst a set of numbers or figures
2. Tinker: to play around with numbers, figures, or other mathematical expressions in order to learn something more about them or the situation; experiment
3. Work backwards: to reverse a process as a way of trying to understand it or as a way of learning something new; to work a problem backwards as a way of solving
4. Change or simplify the problem: change some variables or unknowns to numbers; change the value of a constant to make the problem easier; change one of the conditions of the problem; reduce or increase the number of conditions; specialize the problem; make the problem more general
5. Take things apart: to break a large or complex problem into smaller chunks or cases, achieve some understanding of these parts or cases, and rebuild the original problem; to focus on one part of a problem (or definition or concept) in order to understand the larger problem
6. Re-examine the problem: To look at a problem slowly and carefully, closely examining it and thinking about the meaning and implications of each term, phrase, number and piece of information given before trying to answer the question posed.
7. Check for plausibility: To routinely check the reasonableness of any statement in a problem or its proposed solution, regardless of whether it seems true or false on initial impression; to be particularly skeptical of results that seem contradictory or implausible, whether the source be peer, teacher, evening news, book, news- paper, internet or some other; and to look at special and limiting cases to see if a formula or an argument makes sense in some easily examined specific situations
8. Prove: to desire that a statement be proved to you or by you; to engage in dialogue aimed at clarifying an argument; to establish a deductive proof; to

use indirect reasoning or a counter-example as a way of constructing an argument (Asdourian, et al., 2011)

The problems that I will ask students to attempt are challenging. My goal is to make students comfortable with the idea that there are many different ways at arriving at a correct answer and, often, many correct answers. I also hope to instill in students the belief that struggle is not failure, and that creativity and persistence are skills that improve only with time and practice.

Strategies

I anticipate the most resistance to the idea of "productive struggle." It is uncomfortable to make mistakes, difficult to think in ways to which we are not accustomed, and frustrating to attempt something and fail (sometimes repeatedly). In order to create an environment where these things do not stymy student effort, I will work to establish a supportive community of learners. At the beginning of the school year, I will give groups hypothetical classroom situations that involve struggle (productive or otherwise) and I will ask students in each class to develop their own norms and expectations of each other that they believe will make it easiest for them to struggle, make mistakes, and learn. Additionally, I will acknowledge and frequently give counterexamples to common stereotypes to help students debunk negative beliefs they may hold both about their own and others' mathematical abilities.

I plan to seat my students in groups and will ask students to model effective and respectful collaboration at the beginning of the school year. I will incentivize collaboration by making it a part of a weekly participation grade. I recognize that participation takes different forms for different students and will search for any form of active engagement – ex. quieter students may share ideas in writing or diagrams, more advanced students may take on a role of explaining to students who struggle, students may ask questions of each other and of other groups. In addition to modeling collaboration, I will also identify clear examples of students using any of the mathematical habits of mind (ideally, without pre-teaching of those habits) and encourage students to discuss of how that habit could be applied to a given problem or otherwise. Lastly, I will provide as many manipulatives and hands-on activities as possible, because I've noticed that freshmen in particular are immeasurably more engaged with the material when they are given a tool to concretely manifest a mathematical concept.

I want to foster the idea of more than one correct answer and encourage students to try to view the problems that they're solving from multiple perspectives. Independent of this specific unit, during the introductory activity of each class, I will provide prompts that allow and encourage students to keep in mind both of the above maxims. The activities will largely be selected from the following resources, which will also be listed on my sources page: Which One Doesn't Belong, Would You Rather, Estimation 180, Visual Patterns, Graphing Stories, Math Mistakes, 101 questions, Open Middle. For these prompts, there are often multiple – if not infinite – ways in which students can answer and experience success. This will be a useful tool in creating a culture where there is room for multiple interpretations and understandings of a problem, so that all students feel more comfortable participating and taking mathematical risks.

Classroom Activities, Resources, Appendices

I chose problems for this unit that I intend to use throughout the year. In addition to the main problems, I've chosen a number of more challenging examples of questions that utilize essentially the same habit of mind, but may require more creativity, more calculations, or a greater mastery of abstract generalities. The two main sources from which I took these problems are from the Park School Math Curriculum and from Boris Kordemsky's *The Moscow Puzzles,* but this unit can easily be adapted to include problems from a wide variety of sources, as long as they require or encourage students to use the earlier-identified habits of mind.

In order to assess students' growth in both content and skill, I will include one problem similar to those identified below on bi-weekly assessments that students must complete independently. On these tested questions, I will use a rubric to assess students' attempts to persevere, develop a strategy, test that strategy, use different strategies if necessary, and attempt an answer. Ideally, students will answer the question correctly, but my focus will be primarily on students' approach to the problem and the documentation of their thinking. At the end of the first quarter, students will have a day to work with their table groups to solve a single problem. I will ask them to give a written explanation for their work, a reflection on the strategies they used and their effectiveness, and proof that their solution makes sense (or proof that it doesn't if they weren't able to arrive at a feasible solution in the given time.) The written explanation can be an essay, a mind-map, a flowchart, a diagram, or other teacher-approved format that involves an explanation that the students and their peers can make sense of. Students will grade each other's presentations and rate each other as contributing partners in the group. I will repeat this activity for the second quarter and ask student groups to present their problem, work, and solution to the class. I will assess student work prior to the presentation, but students will have a chance to independently correct or add to their answers before they present to their peers. For the third and fourth quarter, students groups will present as well, but groups will change each quarter. In each quarter, I will attempt to focus on one or more specific habits of mind. However, throughout the entire school year, I will repeatedly emphasize the importance of the following three habits of mind; Re-examine the problem, Check for plausibility, Prove it. In the following pages, I have provided a variety of lessons that can be used in middle or high school level classes.

<u>Problem 1: Card Magic</u>
Habits of Mind: Tinker, Look for Patterns, Change or Simplify the Problem

Materials: Decks of cards, dice for extension

Prompt: In a well-shuffled 52-card deck, half the cards are red and half are black. If the number of red cards in the top half of the deck is added to the number of black cards in the bottom half of the deck, the sum is 30. How many red cards are in the top half? (Parkmath Book 1)

Extension: Dice problem: What is the sum of the covered faces of this stack of dice? (I will provide an image for students). Student groups (who finish the prior problem early) will be given a single dice to work with for this question. They will have to notice that the opposite side of the dice always sum to 7 and will be able to use this to determine the sum of the covered faces in the image provided.

Guiding Questions:

- This seems like an overwhelming task. BUT, each group has two decks of cards on their desks. What could you do? (experiment: shuffle and count)
- Ok, this process takes a LONG time. How could you simplify it? (make the deck smaller with half red and half black cards and experiment)
- What patterns are you noticing? Why do you think those are the case?
- Is there an exception to the patterns?

Solution and Discussion:
Solution: There are 15 red cards in the top half of the deck. Whatever cards that are NOT red in the top 26 cards of the deck are black. So 26 – red = black AND for the bottom half 26 – black = red. For example, if you have 12 red cards in the top half, that means you would have to have 26 – 12 = 14 black cards in the top half. That means that in the bottom 26, there would have to be 12 black cards so that the black cards in the top half of the deck (14) and the black cards in the bottom half of the deck (12) sum to half the deck (26). Whatever number of red cards there are in the top half of the deck is equal to the number of black cards that are in the bottom half of the deck, so if the number of red cards from the top of the deck summed with the number of black cards from the bottom of the deck is 30, then the red in the top must be 15 and the black in the bottom must be 15.

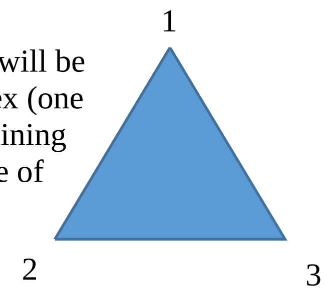
Discussion: Don't believe this? Test it.
Post-it boards – What was helpful when you were working in your groups? What WASN'T helpful? (reinforce productive, supportive norms for working in groups).
Did you have any strategy that you felt was really useful in helping your solve this problem? What would you do differently if you had to do it again?

Problem 2: Magic Triangle

Habits of Mind: Tinker, Look for Patterns, Change or Simplify the Problem, Check for Plausibility

Materials: You could provide the initial magic triangle on sheets for each individual, but the image is simple enough that a projection would be sufficient

Prompt: Magic Triangle (The Moscow Puzzles, pg 8, #22). Students will be given an image of a triangle with the numbers 1, 2, and 3 at each vertex (one per vertex). They will then have to determine how to arrange the remaining digits 4, 5, 6, 7, 8, and 9 (each can be used only once) so that each side of the triangle sums to 17.



Extension: How would this change if, without being told which numbers to place at the vertices, you had to make a similar arrangement of the numbers 1 through 9 that summed to 20 on each side?

Guiding Questions: Students will immediately start filling in numbers and tinkering. However, it might be helpful to remind students that the side with 1 and 2 will need to use larger numbers than the side containing 2 and 3 or the side containing 1 and 3. I might ask if they notice a pattern about the sums of the vertices (the side with the smallest sum always gets the largest possible number).

- Which combinations can you rule out? (anything that can't possibly add up to 17, or to 20 for the extension)
- For the extension: Which numbers in these arrangements are most important? (those at the vertices – starting by choosing those numbers might be helpful, recognizing that they can't possibly be 1, 2, and 3 and ruling out other groupings as they go)

Solution and Discussion:

Solution: For the first triangle, a side would contain 1 9 5 2, a second side would contain 2 4 8 3, and the third side would contain 3 6 7 1. For the extension, there are multiple solutions, but one is as follows: 1 8 6 5, 5 2 4 9, 9 7 3 1.

Discussion: There is only one possible solution for the first prompt, but I'm anticipating that at least one group will have noticed that the 9 *has* to go on the side of the triangle containing the 1 and the 2 and that they were able to guess and check from there. For the extension, I would look for a response around balancing the values of the vertices – for the solutions, you could use the lowest (1), middle (5), and highest number (9) at the vertices; you could use the three middle numbers (4, 5, 6) at each vertex, and you could use the middle number in each group of three low (2), middle (5), and high (8). I would also welcome discussion of any other strategy that students found particularly useful.

<u>Problem 3: Figure out the missing letters</u>
Habits of Mind: Tinker, Take the problem apart, Re-examine the problem

Prompt: Each letter in the addition problem below represents a different number. What are the only possible values of A, B, C, and D? Prove it.

$$\begin{array}{r} AB \\ +\ BC \\ \hline ADD \end{array}$$

Extension: THIS + IS = HARD (Parkmath, Book 1, pg 12 #17), Multiple Addition, Summing It Up 2, Division in Disguise (The Great Book of Math Teasers, pgs 44, 46, and 47) – these are slightly more complex iterations of the problem above, and would require greater thought and a more nuanced understanding of how to "take the problem apart."

Guiding Questions:

- What do you notice about AB and BC? (They're two digit numbers)
- How does this help you start to limit the possible answers? (Because no sum of any two digit numbers could be greater than 199, so A must be 1).
- What does that mean about A and B? (Their sum must be greater than 9 because a hundreds place was created.)
- Why doesn't B = 9 work? (because then C and D would both have to have a value of 0 and each different letter has to have a different numeric value)

I anticipate that student groups will finish the first problem with sufficient time remaining in class to attempt at least one of the extension options. The first two are very similar to the original problem, but the second two might require some clarification.

- Summing it up: clarify that each different symbol represents a different number. What could you do to start? (try different combinations of numbers that sum to 13)
- Division in Disguise: Do you notice anything that you think would be very important? (there is NO remainder, so the two numbers must be evenly divisible)

Solution and Discussion:
Solution: A = 1, B = 8, C = 2 and D = 0
Discussion: If you were completely stuck on this problem, how did you start? Ideally, students would answer that they just started to try numbers and then noticed that the sum of the two numbers AB and BC would have to be greater than 99 and less than 200 in order to get a possible three digit answer. I would look for student responses to explain how they were able to rule out digits (check for plausibility) to narrow down answers.

<u>Problem 4</u>
Habits of Mind: Tinker, Look for Patterns, Check for Plausibility

Prompt: The symbol ¤ takes a single number, squares it, and then subtracts 4.

a. What is ¤(6)?
b. Can you ever get a negative answer for ¤(*x*)? Why or why not?
c. Find an *x* so that ¤(*x*) is divisible by 5

Extension: 1. Let $x@y=x^2-y^2$ .For example 4@3 = 16 – 9 = 7. True or false: *x*@*y* always equals the sum of the two numbers – for example 4@3 = 7 which equals 4 + 3. If it's true, justify your claim. If it's false, explain what kinds of numbers do make the claim work.
2. Tinker to find a rule for *x* # *y* that gets the following answer: 5#1=24. Then try to write a rule that gives 5#1 = 24 *and* 4#2 = 14

Guiding Questions:

- I anticipate that students will immediately be intimidated by the unfamiliar symbols. I might ask them about other symbols they know of (and eventually get to the idea that math is all symbols, these are just different than the ones we normally see)
- How can you prove your answer for part b? (plug in numbers! Do you see a pattern? Does the pattern stay the same forever or is there a point where it starts to change a little bit?)
- For the extension: What would make it true? (it would have to work for ALL possible number combinations, which should immediately be evident to be impossible)
- Extension: If it's false, when does it work? What do you notice about the example given? What other pair of numbers might be similar enough to 4 and 3 that they would produce these results

Solution and Discussion:
Solution: a) ¤6 = 6^2 – 4 = 32 b) Yes, only when $x^2 < 4$, so when x = -1, 0, or 1 c) ¤8 = 8^2 – 4 = 60 or ¤7 = 49 – 4 = 45 There are multiple answers for this.
Extension 1: False! This only works for two consecutive numbers where the greater number is x. Ex 9@8 = $9^2 - 8^2$ = 81 – 64 = 17 and 9 + 8 = 17. But if you use non-consecutive numbers or plug the greater number in for y, this no longer works.
Extension 2: 5#1 = 24 → Lots of possible answers 4x + 4y, $x^2 - y$, $(xy)^2 - y$, 4(x + y). BUT, if you want the rule to work for 4#2 = 14, then $x^2 - y$ is the only rule that works
Discussion: For the first prompt, I would aim to discuss trial and error methods that students used OR any sort of logical shortcuts that they used to spend less time on those methods. For the second two prompts, I would again ask students to detail their methods of arriving at an answer but also probe for any pattern-recognition that students used.

Example Scoring Rubric for Problem Solving:
http://educationnorthwest.org/sites/default/files/scoregrid.pdf

Common Core State Standards for Mathematics
CC.2.2.7.B.3
Model and solve real-world and mathematical problems by using and connecting numerical, algebraic, and/or graphical representations.

CC.2.2.HS.D.1
Interpret the structure of expressions to represent a quantity in terms of its context.

CC.2.2.HS.D.2
Write expressions in equivalent forms to solve problems.

CC.2.2.HS.D.8
Apply inverse operations to solve equations or formulas for a given variable.

CC.2.2.HS.D.9 Use reasoning to solve equations and justify the solution method.

CC.2.2.HS.D.10
Represent, solve and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.

CC.2.1.HS.F.2
Apply properties of rational and irrational numbers to solve real world or mathematical problems.

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Teacher Resources

Estimation 180: http://www.estimation180.com/ This website offers useful prompts that require students to estimate seemingly difficult quantities. It has a lot of visuals and helps inspire conversation.

Robert Kaplinsky: https://robertkaplinsky.com/ This teacher website has a lot of prompts that are similar to those found on the rest of the sites on this list

Visual Patterns: http://www.visualpatterns.org/teachers.html This website offers prompts that require students to recognize patterns visually, which is particularly useful for students who are intimidated or turned off by math.

Graphing Stories: http://www.graphingstories.com/ These video stories are *very* useful in helping students translate from a situational context to a graphical one and really help to make clear the relationship between the quantities measured on the x and y axes and what is happening in each of the stories.

Math Mistakes: http://mathmistakes.org/ **I usually use my own students' mistakes, but this site is useful if you want to review specific types of mistakes and don't have an example on hand

101 questions: http://www.101qs.com/ These are very open-ended and it's sometimes a little harder to get conversation headed in a mathematical direction when using these prompts, but with a little encouragement, students can really use these to start asking great mathematical questions.

Would You Rather: http://www.wouldyourathermath.com/ This site is excellent because it forces students to really justify their answers and even those students who struggle in math can easily make arguments for which option they would rather chose, so it gets everyone participating.

Which One Doesn't Belong: http://wodb.ca/ Also a great option to get everyone participating because there are no wrong answers, and they only involve recognition of shared characteristics.

Open Middle: http://www.openmiddle.com/category/high-school-algebra/ These are more challenging problems that are useful because there are multiple ways to approach and solve each of them. There is usually a single correct answer, but students can get to those answers in a variety of different ways.

Abstract

Unless presented frequently with opportunities to practice habits of mind that facilitate non-standard problem solving, students focus so much on performing a process to get a “right” answer, that they neglect to think about what the answer means in the context of a problem, and whether the numerical answer(s) at which they’ve arrived really “makes sense” considering what a prompt or question is asking of them. They also struggle to draw connections between different topics throughout the duration of a math course and this inability to see relatedness among various topics hinders their ability to really grasp mathematical content in a way in which they can effectively argue about the reasoning used to solve a problem and arrive at an answer. And, as many math teachers have experienced, if students see a problem that looks different than those they’ve practiced over and over again in class, they often shut down. This unit is designed to enable students to approach these unfamiliar and challenging problems with a greater degree of confidence and a greater ability to put to use skills they all already have to successfully solve these problems. I want students’ thought processes to involve a reaction to affirm that their reasoning is sound with relation to a problem, and also use that same process to question their own reasoning and the reasoning of others if the answers or justifications by which they or others have reached an answer seem questionable or unsound. If students are better able to recognize when and why answers don’t make sense, and they have the confidence to explain this, they will have a far easier time of correctly identifying whatever mistake or mistakes were made in the process of arriving at an incorrect answer, and will be able to more effectively rectify those mistakes. When students realize the tools that they’ve always had at their disposal can be used to solve problems that would have previously made them cringe, it can be a game-changer, and can help students develop a genuine curiosity about mathematics and a desire to learn, rather than a persistent fear of and disinterest in the subject.